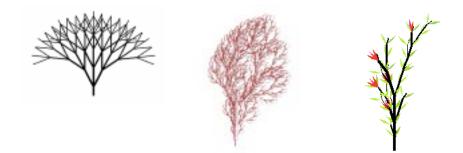
CPS 140 - Mathematical Foundations of CS Dr. S. Rodger Section: Modeling using L-Systems (handout)



L-system

L-systems are grammatical systems introduced by Lyndenmayer to describe biological developments such as the growth of plants and cellular organisms.

An L-system is composed of three parts (Σ, h, w)

Σ	finite alphabet	set of symbols
h	rewriting rules	each symbol is replaced by string of symbols
w	axiom	starting point

h is finite substitutions, $h: \Sigma \to \Sigma^*$.

h(w) is computed by replacing every symbol in w that has a rewrite rule by that rule.

A language L of an L-system is the word sequence generated by

- $h^0(w) = w$
- $h^1(w) = h(w)$
- $h^2(w) = h(h(w))$
- ...

 $\mathcal{L} = \{ h^i(w) \mid i \ge 0 \}$

NOTE: If h(a)=bb we will write this as a rule

$$a ==> bb$$

meaning the symbol a can be replaced by the symbols bb.

Example:

```
 \begin{array}{ll} \Sigma & \text{alphabet: } \{a,b\} \\ \text{h} & \text{rules:} & \text{a} ==> \text{a} \\ & & \text{b} ==> \text{ab} \\ \text{w} & \text{axiom:} & \text{ab} \end{array}
```

Notes:

- = means "is replaced by"
- left hand side of rule must be a single character
- there is at most one rule for each character

What is the language L of strings represented by this L-system? that is, starting with the axiom, what are all the strings that can be generated from the rules?

L =

Example:

```
\Sigma alphabet: {a}
h rules: a ==> aa
w axiom: a
```

L =

Drawing a picture of an L-system

Defining an L-system: (3 parts in this order)

- Axiom definition: This must be the first line of the file
- Production rules: Defines the replacement rules. There must be spaces between the symbols on the right hand side of rules.
- Geometric rules: Defines colors, widths, etc.

Symbols for drawing and moving:

- g: draw a line one step in the current direction
- f: move forward one step in the current direction

Example:

This example is in the file: samp1

-	
SET axiom X	axiom definition
X ==> g f g	production rule
SET d 15 SET iniwidth 5 SET color black	geometric rules length of line drawn is 15 units width of initial line is 5 units

Example:

This example is in the file: samp2

L
I
l
l
l
I
ı

SET axiom X	start symbol is X
X ==> g f g X	only change from previous program, repetition
SET d 15 SET iniwidth 5 SET color black	length of line width of initial line



Symbols for changing direction

- +: change direction to the right in a determined angle
- $\bullet~-:$ change direction to the left in a determined angle
- &: change direction pitch down in a determined angle
- \wedge : change direction pitch up in a determined angle
- *: change direction roll left in a determined angle
- /: change direction roll right in a determined angle
- %: change direction 180 degrees

Example:

This example is in the file: samp3

Fre

SET axiom X	start symbol is X
$\begin{array}{l} X ==> g g g X + Y \\ Y ==> g \end{array}$	rule with 2 replacements
SET d 15 SET iniwidth 5 SET angle 15 SET color blue	length of line width of initial line angle for change of direction initial color

L =

Example:

We will make just a slight change in the L-system. This example is in the file: samp4

X ==> g change blue g g change yellow + Y X Y ==> g

SET d 15 SET iniwidth 5 SET angle 15 SET color black

SET axiom X

only difference with file samp3

length of line width of initial line angle for change of direction initial color

L =

Stacking operations

- [: save in stack status of turtle which is current direction position and width of line
-]: recover from stack status of turtle

To make a branch, the turtle must draw one part of the branch and then come back to the fork position and draw the other part of the branch. Part of a string can be saved for processing by putting it within brackets [].

Example, consider the rules

SET axiom X $X ==> g [\sim + Y g] g$ within []'s is a branch
the ~ means decrement the width of the lineY ==> + YSET d 18SET iniwidth 4sET incwidth 1SET angle 30increment for changing width of lineSET color blackset of the set of

First string in L is $g[\sim+Yg]g$. To draw this first draw the first line for the first g. At this point, save the $[\sim+Yg]$ along with the current direction and the current width of the line. Continue drawing at the first symbol past the]. Draw a line. Now that the end of the string has been reached, come back to the point in the drawing where the branch occured and draw the string within the []'s.

What is L? L =



Suppose we change the X rule above by adding X onto the end:

$$X ==> g [\sim + Y g] g X$$

Now the L-system looks like:



Example of drawing plants via L-systems

Now we will examine some examples of growing plants.

Example:

This is in file: plant1

SET axiom X

$$\begin{split} &X ==> g \ [\ \sim + g \ Y \] g \ X \\ &Y ==> g \ g \ [\ \sim + g \ leaf \] \\ &leaf ==> [\ color \ Green \ \{ + \ f - f \ f - f \ + \ \% \ + f - f \ f - f \ \} \] \end{split}$$

Differences from previous examples:

- The color field allows you to specify a color for part of the drawing.
- The parenthesis { } are used to define a region that is to be filled in.

Example:

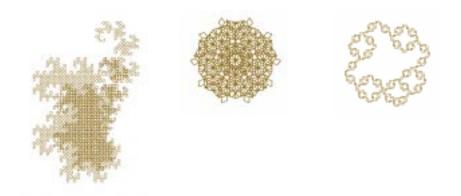
This is in file: plant2



SET axiom X

```
\begin{split} \mathbf{X} &==> \mathbf{g} \left[ \, \sim + \mathbf{g} \, \mathbf{Y} \, \right] \left[ \, \sim - - \mathbf{g} \, \mathbf{Y} \, \right] \mathbf{g} \, \mathbf{X} \\ \mathbf{Y} &==> \mathbf{X} \, \mathbf{g} \, \mathbf{g} \left[ \, \sim + \mathbf{g} \, \mathrm{leaf} \, \right] \\ \mathrm{leaf} &==> \left[ \, \mathrm{color} \, \mathrm{Green} \, \left\{ + \, \mathbf{f} - \mathbf{f} \, \mathbf{f} - \mathbf{f} \, + \, \% \, + \, \mathbf{f} - \, \mathbf{f} \, \mathbf{f} - \, \mathbf{f} \right\} \, \right] \\ \mathrm{SET} \, \mathbf{d} \, \mathbf{18} \\ \mathrm{SET} \, \mathrm{iniwidth} \, \mathbf{4} \\ \mathrm{SET} \, \mathrm{incwidth} \, \mathbf{1} \\ \mathrm{SET} \, \mathrm{angle} \, \mathbf{18} \\ \mathrm{SET} \, \mathrm{color} \, \mathrm{black} \end{split}
```

Fractals



References:

- The Algorithmic Beauty of Plants, by P. Prusinkiewicz and A. Lindenmayer
- Automata, Languages, Development, by A. Lindenmayer and G. Rozenberg