CPS 270: Assignment 3

Due Midnight, March 6th
Please submit a PDF of your solutions via Sakai.

Question 1

Show that both of De Morgan’s Laws are true, using truth tables. [10]

Question 2

Derive the truth value of $A$, given the following logical facts in a Knowledge Base:

$B \Rightarrow \neg C$
$A \lor B$
$C \lor D$
$\neg D \land E$. [10]

Question 3

You are given the following predicate vocabulary, where $x$ and $y$ are parameters:

AuthorOf(x, y)
IsAuthor(x)
IsProgram(x)
Equals(x, y).

Write down first-order logic sentences to express the following:

1. All programs have an author.
2. RichardStallman wrote the program emacs.
3. At least one program has exactly three authors.
4. No programs were written by George. [10]
**Question 4**

You are given the following joint probability table for binary random variables $A$, $B$, and $C$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>True</td>
<td>True</td>
<td>0.288</td>
</tr>
<tr>
<td>True</td>
<td>True</td>
<td>False</td>
<td>0.192</td>
</tr>
<tr>
<td>True</td>
<td>False</td>
<td>True</td>
<td>0.072</td>
</tr>
<tr>
<td>True</td>
<td>False</td>
<td>False</td>
<td>0.048</td>
</tr>
<tr>
<td>False</td>
<td>True</td>
<td>True</td>
<td>0.060</td>
</tr>
<tr>
<td>False</td>
<td>True</td>
<td>False</td>
<td>0.140</td>
</tr>
<tr>
<td>False</td>
<td>False</td>
<td>True</td>
<td>0.060</td>
</tr>
<tr>
<td>False</td>
<td>False</td>
<td>False</td>
<td>0.140</td>
</tr>
</tbody>
</table>

Answer the following questions (showing all working):

1. Are $C$ and $A$ independent?

2. Is $B$ independent of $A$ given $C$?

[20]

**Question 5**

You are given the following Bayes Net.

Write down the CPTs you would need to fully describe this Bayes Net, and their sizes. (You do not need to fill in the CPTs, just write down what expression they represent, and the number of entries they have.)  

[10]
Question 6

You are given the following Bayes Net:

\[ A \rightarrow B \rightarrow C \]

with \( P(A = \text{True}) = 0.6 \), and the following CPTs:

\[
\begin{array}{c|cc}
B & A & P \\
\hline
\text{True} & \text{True} & 0.6 \\
\text{False} & \text{True} & 0.4 \\
\text{True} & \text{False} & 0.1 \\
\text{False} & \text{False} & 0.9 \\
\end{array}
\]

\[
\begin{array}{c|cc}
C & B & P \\
\hline
\text{True} & \text{True} & 0.25 \\
\text{False} & \text{True} & 0.75 \\
\text{True} & \text{False} & 0.5 \\
\text{False} & \text{False} & 0.5 \\
\end{array}
\]

(a)

(b)

Draw 3 samples from the joint distribution. Show your working. [10]