CPS 1: Problem Solving and Algorithms

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Problem

- Precise description and information
- Permissible operations and actions
- Goal – defines solution to problem

Methodology

- Understand
- Think of approach
- Implement
- Verify
What does the problem mean?

- Read problem **at least 2 times**
  - first time, get an overall sense of problem
  - second read, focus on finer details
- Restate the problem
- Can you describe the problem with a figure?
- Identify knowns and unknowns
- Identify constraints
What approach should I take?

- Look for relation b/w knowns and unknowns
- Break it down into subproblems that I can already solve: “Divide and Conquer”
- Compare to a similar problem that I have solved previously
- Solve an easier version
- Guess solution and work backwards
Implementation

- Perform calculations
- Solve equations
- Find a solution
- **Test each step**: convince yourself that each step is correct
Verification

- Does my solution make sense?
- Does it work for the original problem?
- Have I used all the information I was given?

**Example**: The sum of two consecutive integers is 23. Find these integers.
Solution

- **Understand:**
  known – sum is 23
  unknowns – the consecutive integers \( x \) and \( x+1 \)

- **Approach:**
  \( x+(x+1)=23 \)

- **Implement:**
  \( x=11, \ x+1=12 \)

- **Verify:**
  \( 11+12=23 \)
Problem Statement: Three consecutive odd integers add up to 57. Find these integers.

- Similar to previous problem
  - 3 unknowns, instead of 2
  - integers are odd
- Representation: $x, x+2, x+4$
- Approach: $x+(x+2)+(x+4)=57$
- Solve and check: $x=17, x+2=19, x+4=21$
After gathering 770 chestnuts, the three little girls divided them up so that their amounts were in the same proportion as their ages. As often as Mary took four chestnuts, Nellie took three, and for every six that Mary received, Susie took seven. How many chestnuts did each girl get?
Solving the problem

- After gathering 770 chestnuts, the three little girls divided them up so that their amounts were in the same proportion as their ages. As often as Mary took four chestnuts, Nellie took three, and for every six that Mary received, Susie took seven. How many chestnuts did each girl get?

- Notice that Mary is related to both Susie and Nellie
  - Let $c = \text{number of chestnuts Mary takes}$
  - Then $\frac{3c}{4} = \text{number of chestnuts Nellie takes}$
  - $\frac{7c}{6} = \text{number of chestnuts Susie takes}$
  - As there are 770 chestnuts in all,
    $$c + \frac{3c}{4} + \frac{7c}{6} = 770$$

- Solve and check
Problem solving is easier with practice

“Solving problems is a practical art, like swimming, or skiing, or playing the piano; you can learn it only by imitation and practice ... if you wish to learn swimming you have to go into the water, and if you wish to become a problem solver you have to solve problems.”

G. Polya
A more CS example

**Twenty Questions**

“I'm thinking of a number between 1 and 1,000,000. Your task is to guess it... using at most 20 questions. You are only allowed to ask questions that require a "yes" or "no" response. Come up with an algorithm for asking questions that will find the number regardless of which one I pick.”
What is an “Algorithm”? 

- Sequence of precise steps, with a termination criterion, that solves a problem 
- *Correct, complete, precise, and it terminates.* 
- Correct: returns the right answer 
- Complete: always returns an answer 
- Correct and Complete: always returns the right answer 
- Precise: steps are clear-cut and easy to execute 
- Good algorithms are efficient, with as few steps as possible 
- Good algorithms minimize resource usage (such as memory space)
Shampoo algorithm

Q: Why did the computer scientist die in the shower?
A: He followed the instructions on the shampoo bottle.

Problem with shampoo algorithm:
- no terminating condition!
Shampoo algorithm has what is called an infinite loop
How to fix?
Describing an Algorithm

Pictures
Plain English
Pseudo-code (almost a programming language)
Programming Language

More easily expressed

More precise
Central role of algorithms in CS

- Limitations of Algorithms
- Execution of Algorithms
- Analysis of Algorithms
- Discovery of Algorithms
- Communication of Algorithms
- Representation of Algorithms
Components of an Algorithm

- **Data structures**: hold information/data
- **Instructions/Operations**: change data values within data structures
- **Conditional Expressions**: make decisions
- **Control Structures**: act on decisions