Hashing: Log (10^{100}) is a big number

- Comparison based searches are too slow for lots of data
  - How many comparisons needed for a billion elements?
  - What if one billion web-pages indexed?

- Hashing is a search method: average case O(1) search
  - Worst case is very bad, but in practice hashing is good
  - Associate a number with every key, use the number to store the key
    - Like catalog in library, given book title, find the book
  - A hash function generates the number from the key
    - Goal: Efficient to calculate
    - Goal: Distributes keys evenly in hash table

Hashing details

- There will be collisions, two keys will hash to the same value
  - We must handle collisions, still have efficient search
  - What about birthday "paradox": using birthday as hash function, will there be collisions in a room of 25 people?

- Several ways to handle collisions, in general array/vector used
  - Linear probing, look in next spot if not found
    - Hash to index h, try h+1, h+2, ..., wrap at end
    - Clustering problems, deletion problems, growing problems
  - Quadratic probing
    - Hash to index h, try h+1^2, h+2^2, h+3^2, ..., wrap at end
    - Fewer clustering problems
  - Double hashing
    - Hash to index h, with another hash function to j
      - Try h, h+j, h+2j, ...

Chaining with hashing

- With n buckets each bucket stores linked list
  - Compute hash value h, look up key in linked list table[h]
  - Hopefully linked lists are short, searching is fast
  - Unsuccessful searches often faster than successful
    - Empty linked lists searched more quickly than non-empty
  - Potential problems?

- Hash table details
  - Size of hash table should be a prime number
  - Keep load factor small: number of keys/size of table
  - On average, with reasonable load factor, search is O(1)
  - What if load factor gets too high? Rehash or other method

Hashing problems

- Linear probing, hash(x) = x, (mod table size)
  - Insert 24, 12, 45, 14, delete 24, insert 23 (where?)

- Same numbers, use quadratic probing (clustering better?)

- What about chaining, what happens?
What about hash functions

- Hashing often done on strings, consider two alternatives

```java
public static int hash(String s)
{
    int k, total = 0;
    for(k=0; k < s.length(); k++){
        total += s.charAt(k);
    }
    return total;
}
```

- Consider `total += (k+1)*s.charAt(k)`, why might this be better?
  - Other functions used, *always mod result by table size*

- What about hashing other objects?
  - Need conversion of key to index, not always simple
  - Ever object has method `hashCode()`!

Tools: Solving Computational Problems

- Algorithmic techniques and paradigms
  - Brute-force/exhaustive, greedy algorithms, dynamic programming, divide-and-conquer, ...
  - Transcend a particular language
  - Designing algorithms, may change when turned into code

- Programming techniques and paradigms
  - Recursion, memo-izing, compute-once/lookup, tables, ...
  - Transcend a particular language
  - Help in making code work
    - Avoid software problems (propagating changes, etc.)
    - Avoid performance problems

Quota Exceeded

- You’re running out of disk space
  - Buy more
  - Compress files
  - Delete files

- How do you find your “big” files?
  - What’s big?
  - How do you do this?

Recursive structure matches code

```java
public static long THRESHOLD = 1000000L; // one million

public static void findBig(File dir, String tab)
{
    File[] dirContents = dir.listFiles();
    System.out.println(tab+"**:");
    for(File f : dirContents){
        if(f.isDirectory()) {
            findBig(f,tab+"\t");
        } else {
            if(f.length() > THRESHOLD){
                System.out.printf("%s%s%8d",tab,f.getName(), f.length());
            }
        }
    }
}
```

Does `findBig` call itself?
Solving Problems Recursively

- Recursion is an indispensable tool in a programmer’s toolkit
  - Allows many complex problems to be solved simply
  - Elegance and understanding in code often leads to better programs: easier to modify, extend, verify (and sometimes more efficient!)
  - Sometimes recursion isn’t appropriate, when it’s bad it can be very bad—every tool requires knowledge and experience in how to use it

- The basic idea is to get help solving a problem from coworkers (clones) who work and act like you do
  - Ask clone to solve a simpler but similar problem
  - Use clone’s result to put together your answer
- Need both concepts: call on the clone and use the result

Print words read, but print backwards

- Could store all the words and print in reverse order, but ...
  - Probably the best approach, recursion works too

```java
 public void printReversed(Scanner s) {
    if (s.hasNext()) {  // reading succeeded?
        String word = s.next();    // store word
        printReversed(s);          // print rest
        System.out.println(word);  // print the word
    }
}
```

- The function printReversed reads a word, prints the word only after the clones finish printing in reverse order
  - Each clone has own version of the code, own word variable
  - Who keeps track of the clones?
  - How many words are created when reading N words?
    - Can we do better?

Exponentiation

- Computing \( x^n \) means multiplying \( n \) numbers (or does it?)
  - What’s the simplest value of \( n \) when computing \( x^n \)?
  - If you want to multiply only once, what can you ask a clone?

```java
 public static double power(double x, int n) {
    if (n == 0) {
        return 1.0;
    }
    return x * power(x, n-1);
}
```

- Number of multiplications?
  - Note base case: no recursion, no clones
  - Note recursive call: moves toward base case (unless ...)

Faster exponentiation

- How many recursive calls are made to compute \( 2^{1024} \)?
  - How many multiplies on each call? Is this better?

```java
 public static double power(double x, int n) {
    if (n == 0) {  
        return 1.0;
    }
    double semi = power(x, n/2);  
    if (n % 2 == 0) {
        return semi*semi;
    }
    return x * semi * semi;
}
```

- What about an iterative version of this function?
Back to Recursion

- **Recursive functions have two key attributes**
  - There is a *base case*, sometimes called the *exit case*, which
does not make a recursive call
  - See print reversed, exponentiation
  - All other cases make a recursive call, with some parameter
    or other measure that decreases or moves towards the base
    case
    - Ensure that sequence of calls eventually reaches the base case
    - “Measure” can be tricky, but usually it’s straightforward

- **Example: finding large files in a directory (on a hard disk)**
  - Why is this inherently recursive?
  - How is this different from exponentiation?

Recognizing recursion:

```java
public static void change(String[] a, int first, int last)
{
    if (first < last) {
        String temp = a[first]; // swap a[first], a[last]
        a[first] = a[last];
        a[last] = temp;
        change(a, first+1, last-1);
    }
}
// original call (why?): change(a, 0, a.length-1);
```

- What is base case? (no recursive calls)
- What happens before recursive call made?
- How is recursive call closer to the base case?

The Power of Recursion: Brute force

- **Consider the TypingJob APT problem**: What is minimum
  number of minutes needed to type *n* term papers given page
  counts and three typists typing one page/minute? (assign
  papers to typists to minimize minutes to completion)
  - Example: {3, 3, 3, 5, 9, 10, 10} as page counts
  - How can we solve this in general? Suppose we're told that
    there are no more than 10 papers on a given day.
    - How does this constraint help us?
    - What is complexity of using brute-force?

Recasting the problem

```java
// @return min minutes to type papers in pages
int bestTime(int[] pages)
{
    return best(pages, 0, 0, 0, 0);
}
```

- Instead of writing this function, write another and call it
  ```java
  int best(int[] pages, int index, int t1, int t2, int t3)
  {
      // returns min minutes to type papers in pages
      // starting with index-th paper and given
      // minutes assigned to typists, t1, t2, t3
  }```

Recursive methods sometimes use extra parameters; helper methods set this up
Recursive example 1

double power(double x, int n) 
// post: returns x^n 
{ 
    if (n == 0) 
    { 
        return 1.0; 
    } 
    return x * power(x, n-1); 
}

Recursive example 2

double fasterPower(double x, int n) 
// post: returns x^n 
{ 
    if (n == 0) 
    {   return 1.0; 
    } 
    double semi = fasterPower(x, n/2); 
    if (n % 2 == 0) 
    {   return semi*semi; 
    } 
    return x * semi * semi; 
}

Recursive example 3

String mystery(int n) 
{ 
    if (n < 2) 
    { 
        return "" + n; 
        WriteBinary(-n); 
    } 
    else 
    { 
        return mystery(n / 2) + (n % 2); 
    } 
}