Plan for the week

- Review:
  - Union-Find
- Understand linked lists from the bottom up and top-down
  - As clients of java.util.LinkedList
  - As POJOs
  - Using linked lists to leverage algorithmic improvements
- Binary Trees
  - Just a linked list with 2 pointers per node?
- Self-referential structures and recursion
  - Why recursion works well with linked-structures
- Lab: Boggle
  - Testing & Analysis

Contrast LinkedList and ArrayList

- ArrayList --- why is access O(1) or constant time?
  - Storage in memory is contiguous, all elements same size
  - Where is the 1st element? 40th? 360th?
  - Doesn’t matter what’s in the ArrayList, everything is a pointer or a reference (what about null?)

What about LinkedList?

- Why is access of Nth element linear time?
- Why is adding to front constant-time O(1)?

Linked list applications continued

- If programming in C, there are no “growable-arrays”, so typically linked lists used when # elements in a collection varies, isn’t known, can’t be fixed at compile time
  - Could grow array, potentially expensive/wasteful especially if # elements is small.
  - Also need # elements in array, requires extra parameter
  - With linked list, one pointer used to access all the elements in a collection
- Simulation/modeling of DNA gene-splicing
  - Given list of millions of CGTA... for DNA strand, find locations where new DNA/gene can be spliced in
    - Remove target sequence, insert new sequence
**Linked lists, CDT and ADT**

- As an ADT
  - A list is empty, or contains an element and a list
  - ( ) or (x, (y, ( ) ) )

- As a picture
  ![Linked list diagram]

- As a CDT (concrete data type) pojo: plain old Java object
  ```java
  public class Node
  {
      String info;
      Node next;
  }
  ```

**Building linked lists**

- Add words to the front of a list (draw a picture)
  - Create new node with next pointing to list, reset start of list
  ```java
  public class Node {
      String value;
      Node next;
      Node(String s, Node link){
          value = s;
          next = link;
      }
      // ... declarations here
      Node list = null;
      while (scanner.hasNext()) {
          list = new Node(scanner.next(), list);
      }
  }
  ```

- What about adding to the end of the list?

**Dissection of add-to-front**

- List initially empty
- First node has first word

  ![Dissection diagram]

- Each new word causes new node to be created
  - New node added to front
- Rhs of operator = completely evaluated before assignment

**Standard list processing (iterative)**

- Visit all nodes once, e.g., count them or process them
  ```java
  public int size(Node list){
      int count = 0;
      while (list != null) {
          count++;
          list = list.next;
      }
      return count;
  }
  ```

- What changes in code if we generalize what process means?
  - Print nodes?
  - Append “s” to all strings in list?
Standard list processing (recursive)

- Visit all nodes once, e.g., count them

```java
public int recsize(Node list) {
    if (list == null) return 0;
    return 1 + recsize(list.next);
}
```

- Base case is almost always empty list: null pointer
  - Must return correct value, perform correct action
  - Recursive calls use this value/state to anchor recursion
  - Sometimes one node list also used, two “base” cases
- Recursive calls make progress towards base case
  - Almost always using list.next as argument

Recursion and linked lists

- Print nodes in reverse order
  - Print all but first node and...
    - Print first node before or after other printing?

```java
public void print(Node list) {
    if (list != null) {
        System.out.println(list.info);
        System.out.println(list.info);
        print(list.next);
    }
}
```

Recursion with pictures

- Counting recursively

```java
int recsize(Node list){
    if (list == null)
        return 0;
    return 1 + recsize(list.next);
}
```

```
public void print(Node list) {
    if (list != null) {
        System.out.println(list.info);
        System.out.println(list.info);
        print(list.next);
    }
}
```

Binary Trees

- Linked lists: efficient insertion/deletion, inefficient search
  - ArrayList: search can be efficient, insertion/deletion not

- Binary trees: efficient insertion, deletion, and search
  - trees used in many contexts, not just for searching, e.g., expression trees
  - search in O(log n) like sorted array
  - insertion/deletion O(1) like list, once location found!
  - binary trees are inherently recursive, difficult to process trees non-recursively, but possible
    - recursion never required, often makes coding simpler
From doubly-linked lists to binary trees

- Instead of using prev and next to point to a linear arrangement, use them to divide the universe in half
  - Similar to binary search, everything less goes left, everything greater goes right
- How do we search?
- How do we insert?

Basic tree definitions

- Binary tree is a structure:
  - empty
  - root node with left and right subtrees
- terminology:
  - parent, children, leaf node, internal node, depth, height, path
  - link from node N to M then N is parent of M
  - M is child of N
  - leaf node has no children
  - internal node has 1 or 2 children
  - path is sequence of nodes, N_1, N_2, ..., N_k
  - N_i is parent of N_{i+1}
  - sometimes edge instead of node
  - depth (level) of node: length of root-to-node path
    - level of root is 1 (measured in nodes)
  - height of node: length of longest node-to-leaf path
    - height of tree is height of root
- Trees can have many shapes: short/bushy, long/stringy
  - If height is h, how many nodes in tree?

A TreeNode by any other name...

- What does this look like?
  - What does the picture look like?

```java
public class TreeNode {
    TreeNode left;
    TreeNode right;
    String info;
    TreeNode(String s, TreeNode llink, TreeNode rlink) {
        info = s;
        left = llink;
        right = rlink;
    }
}
```

Printing a search tree in order

- When is root printed?
  - After left subtree, before right subtree.

```java
void visit(TreeNode t) {
    if (t != null) {
        visit(t.left);
        System.out.println(t.info);
        visit(t.right);
    }
}
```
Tree traversals

- Different traversals useful in different contexts
  - Inorder prints search tree in order
    - Visit left-subtree, process root, visit right-subtree
  - Preorder useful for reading/writing trees
    - Process root, visit left-subtree, visit right-subtree
  - Postorder useful for destroying trees
    - Visit left-subtree, visit right-subtree, process root

```
    "llama"
   /     \
"giraffe"  "tiger"
 /   \
"elephant"  "jaguar"
```

Tree functions

- Compute height of a tree, what is complexity?
  ```java
  int height(Tree root)
  {
      if (root == null) return 0;
      else {
          return 1 + Math.max(height(root.left),
                                height(root.right));
      }
  }
  ```

- Modify function to compute number of nodes in a tree, does complexity change?
- What about computing number of leaf nodes?

Balanced Trees and Complexity

- A tree is height-balanced if
  - Left and right subtrees are height-balanced
  - Left and right heights differ by at most one

```
    boolean isBalanced(Tree root)
    {
        if (root == null) return true;
        return isBalanced(root.left) && isBalanced(root.right) &&
               Math.abs(height(root.left) - height(root.right)) <= 1;
    }
```

What is complexity?

- Assume trees are "balanced" in analyzing complexity
  - Roughly half the nodes in each subtree
  - Leads to easier analysis
- How to develop recurrence relation?
  - What is T(n)?
  - What other work is done?
- How to solve recurrence relation
  - Plug, expand, plug, expand, find pattern
  - A real proof requires induction to verify correctness