1. What are the converse and contrapositive of the statement "If it is sunny, then I will go swimming"?
2. Show that $\neg(p \vee \neg q)$ and $q \wedge \neg q$ are equivalent
   (a) using a truth table.
   (b) using logical equivalences.
3. Prove or disprove that $(p \to q) \to r$ and $p \to (q \to r)$ are equivalent.
4. Suppose that $Q(x)$ is the statement "$x + 1 = 2x$". What are the truth values of $\forall x Q(x)$ and $\exists x Q(x)$?
5. Using a truth table, show that $\neg p \vee (q \to p))$ is a tautology.
6. Let $f(n) = 2n + 1$. Is f a one-to-one function from the set of integers to the set of integers? Is f an onto function from the set of integers to the set of integers? Explain the reasons behind your answers.
7. Prove that all the solutions to the equation $x^2 = x + 1$ are irrational.
8. Show that $1^3 + 2^3 + 3^3 + \cdots + n^3$ is $O(n^4)$.
9. Use mathematical induction to show that
$$\sum_{j=0}^{n}(j+1) = \frac{(n+1)(n+2)}{2}$$
whenever n is a nonnegative integer.
10. Prove or disprove that there are six consecutive composite integers.
11. Prove that
$$\sum_{j=n}^{2n-1}(2j+1) = 3n^2$$
whenever n is a positive integer.
12. What is the coefficient of x^2y^7 in $(x + y)^9$?
13. (a) How many functions are there from a set with three elements to a set with eight elements?
    (b) How many one-to-one functions are there from a set with three elements to a set with eight elements?
    (c) How many onto functions are there from a set with three elements to a set with eight elements?
14. Give an asymptotic upper bound for the function $f(n) = (n \log n + n)(n^2 + 1)$.

15. Consider the following relations on $\{1, 2, 3\}$.

$$\begin{aligned} R_1 &= \{(1,1).(1,3),(2,2),(3,1)\} \\ R_2 &= \{(1,1),(2,2),(3,1),(3,3)\} \\ R_3 &= \{(1,2),(2,1),(3,3)\} \\ R_4 &= \{(1,3),(2,3)\} \end{aligned}$$

    (a) Which of these relations are reflexive? Justify your answers.

    (b) Which of these relations are symmetric? Justify your answers.

    (c) Which of these relations are antisymmetric? Justify your answers.

    (d) Which of these relations are transitive? Justify your answers.

16. Suppose that R_1 and R_2 are symmetric relations on a set A. Prove or disprove that $R_1 - R_2$ is also symmetric.

17. Show that every even graph is the union of edge disjoint cycles. (Hint: Use a result shown in recitation 13.)

18. If G is a simple graph in which every vertex has degree at least k, then G contains a path of length at least k. If $k \geq 2$, then G also contains a cycle of length at least $k + 1$. Prove or disprove the two statements.

19. let G_n be the graph whose vertices are the permutations of $\{1, \ldots, n\}$, with two permutations $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ adjacent if they differ by switching two entries. Prove that G_n is bipartite.

20. Embedding $K_{3,3}$ in the plane will always produce a crossing. Show that $K_{3,3}$ can be embedded on a torus without crossings.

21. Prove or disprove that $n^2 - 1$ is composite whenever n is a positive integer greater than 1.

22. Prove that if a and b are even numbers, then $(a - b)^2$ is also even by an indirect proof.

23. Let $f(m, n) = m^n$ be defined for $n, m \in \mathbb{N}$, $n \geq 1$. Define $f(m, n)$ recursively.

24. The function $f^{(k)}(n)$ is defined recursively by

$$f^{(k)}(n) = \begin{cases} n & \text{if } k = 0 \\ f(f^{(k-1)}(n)) & \text{if } k > 0 \end{cases}$$

    where k is an integer. Let $f(n) = n^2$. Find a closed form formula for $f^{(k)}(n)$.

25. Let $H_n = \sum_{j=1}^{n} \frac{1}{j}$. Prove by induction that $H_{2^n} \geq 1 + \frac{1}{2}$, for $n \geq 0$.