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CompSci 102 Discrete Mathematics for CS  
Spring 2005 Forbes In-class Questions 9

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1. A pharmaceutical company is marketing a new test for a certain medical condition. According to clinical trials, the test has the following properties.
  - When applied to an affected person, the test comes up positive in 90% of the cases, and negative in 10%.
  - When applied to a healthy person, the test comes up negative in 80% of the cases with false positives in the remaining 20%.

Suppose that 5% of the US has the condition. When a random person is tested and comes up positive what is the probability that the person actually has the condition?

2. Flip a fair coin, three times. What is the expected number of heads?
3. Suppose  $X$  is a loaded three-sided coin, with possible values  $\{0, 1, 2\}$ , with  $p(0) = 0.2$ ,  $p(1) = 0.5$ , and  $p(2) = 0.3$ . What is  $E(X^2)$ ? What is the variance of the coin?
4. Suppose there are 25 different types of coupons and suppose that each time one obtains a coupon, it is equally likely to be one of 25 types. Compute the expected number of different types that are contained in a set of 10 coupons.
5. A run is a maximal sequence of made shots in a basketball game. For example in the sequence,  $S, S, S, F, S, S, F, F, S$ , where  $S$  represents a made shot (success) and  $F$  represents a missed shot (failure), there are three runs consisting of three made shots, two made shots, and one made shot respectively.

Let  $R$  denote the random variable on the set of sequences of  $n$  independent trials that counts the number of runs in this sequence. Find  $E(R)$ .

Hint: Show that  $R = \sum_{j=1}^n I_j$ , where  $I_j = 1$  if a run begins at the  $j$ th shot and  $I_j = 0$  otherwise. Find  $E(I_1)$  and then  $E(I_j)$ , where  $1 < j \leq n$ .