CPS102 hw1 solutions and hints

February 1, 2007

hw1a Section 1.1, Problem 13 (15 Points)

Solution For day n, the number of pennies you get is

$$\sum_{i=1}^{n} 2^i - 1 = 1 + 2 + 2^2 + ...2^{n-1} = 2^n - 1$$

when $n = 20$, the sum is equal to 1048575. Prove using sum principle.

hw1a Section 1.2, Problem 13 (15 Points)

Solution The number of choices for the favors are 10 choose 2, $C(10, 2)$ (if consider you can have the same flavor, add another 10); the number of choices for the toppings are 3 choose 1, $C(3, 1)$; The total number of choices to choose all, some or none of whipped cream, nuts and a cherry is 2^3. Therefore we have $(C(10, 2) + 10) * C(3, 1) * 2^3$. Note, state your assumptions.

hw1b Section 1.3, Problem 3 (b)(c) (15 Points)

Solution (b)(c) using the Binomial Theorem directly.

$$(x + y)^5 = \sum_{i=0}^{5} \binom{5}{i} x^{5-i} y^i$$

Then expand the above equation. For (c), substitute $y = 2$ in the above equation.

hw1b Section 1.3, Problem 18 (20 Points)

Hints There are two solutions. Solution 1 is based on the fact that the derivative of $(1+x)^n$ is $n(1+x)^{n-1}$, substitute $x = 1$, we get the right hand side (RHS) of the equation we would like to prove. Solution 2 is based on the formula $(1+x)^{n-1}$ times n for the RHS, again substitute $x = 1$. Solution 1:

$$(1+x)^n = \sum_{i=0}^{n} \binom{n}{i} x^i$$

Take derivative of both sides,

$$n(1+x)^{n-1} = \sum_{i=0}^{n} \binom{n}{i} i * x^{i-1}$$

substitute $x = 1$ we get our proof.

hw1c Problem 1(15 Points)

Show that if n is an integer then

$$\sum_{k=0}^{n} 3^k \binom{n}{k} = 4^n$$

Solution Substitute $x = 1$ and $y = 3$ into the binomial theorem.

hw1c Problem 2 (20 Points)

How many ways are there to choose 12 apples from a bushel containing 20 indistinguishable Delicious apples, 20 indistinguishable Macintosh apples, and 20 indistinguishable Granny Smith apples, if at least 3 of each kind must be chosen?

Solution We need to figure out the number of solutions to

$$x_1 + x_2 + x_3 = 12,$$

where $x_i \geq 3$. This is equivalent to the number of nonnegative solutions to

$$y_1 + y_2 + y_3 = 3,$$

where $y_i = x_i - 3$. Using the multi-set formula, where $n = 3$, $k = 3$, we have $C(3+3-1, 3) = 10$.

Remark

You might get points off for not explaining the formula you used. Please talk to me if you have questions regarding your homework / grading.