

# CPS102 Recitation

February 5, 2007

## Problem 1

Show that if  $a, b, c$  and  $d$  are integers such that  $a|c$  and  $b|d$  then  $ab|cd$ .

Solution: since  $a|c$  and  $b|d$ , then we have  $c = as$  and  $d = bt$  for some  $s$  and  $t$ . Multiplying we have  $cd = ab(st)$ , which means  $ab|cd$ .

## Problem 2

Let  $m$  be a positive integer. Show that  $a \bmod m = b \bmod m$  if  $a \equiv b \pmod{m}$ .

Solution: Assume  $a \equiv b \pmod{m}$ . This means that  $m|a - b$ , that is  $a - b = mc$  for some  $c$ . So that  $a = b + mc$ . Let's compute  $a \bmod m$ . We know that  $b = qm + r$  for some  $r < m$ , therefore we have  $a = qm + r + mc = m(q + c) + r$ . By definition, this means that  $a \bmod m = r$ .

## Problem 3

What are the quotient and remainder when

- a) 44 is divided by 8?
- b) -123 is divided by 19?
- c) 1234567 is divided by 1001?
- d) -100 is divided by 101?

- Solution: a) since  $8 * 5 = 40$  and  $44 - 4 = 40$ , we have  $44 \text{ div } 8 = 5$  and  $44 \text{ mod } 8 = 4$ .
- b) We can compute  $123 \text{ div } 19 = 6$  and  $123 \text{ mod } 19 = 9$ . Since the dividend is negative and the remainder is nonzero, the quotient is  $-(6 + 1) = -7$  and the remainder is  $19 - 9 = 10$ . To check, we note that  $-123 = (-7)(19) + 10$ .
- c)  $1234567 \text{ div } 1001 = 1233$  and  $1234567 \text{ mod } 1001 = 334$ .
- d) Since  $100 \text{ div } 101 = 0$  and  $100 \text{ mod } 101 = 100$ , we have  $-100 \text{ div } 101 = -1$  and  $-100 \text{ mod } 101 = 1$ .

## Problem 4

If  $a * 133 - m * 277 = 1$ , does this guarantee that  $a$  has an inverse mod  $m$ ?

Solution: we have  $a * 133 = m * 277 + 1$ , therefore  $a * 133 \equiv 1 \pmod{m}$ ,  $a$  has an inverse  $133 \text{ mod } m$ .

## Problem 5

Solve the following congruence equations for  $x$ . a)  $8x \equiv 1 \pmod{13}$

b)  $99x \equiv 1 \pmod{13}$

Solution:  $x \equiv 5 \pmod{13}$ . and same as a) since  $99 \equiv 8 \pmod{13}$ .