Spatial Decomposition

- Divide the space into *primitive* cells.
- Represent all cells lying in the object.

**Spatial occupation enumeration**
- Divide the space into identical cells arranged in a fixed regular grid structures.
- 3D Analog of 2D images.
- Cells are often cubes and are called *voxels*.
- Popular representation in volume rendering and CAT.
- High storage requirement.
Oct Trees

- Hierarchical representation.
- Requires much less space.
- Extension of 2D *quad tree*.

**Quad tree:**
- Recursively subdivide the plane into four squares by bisecting it in both directions.
- A square is *full, empty, partially full*.
- A partially full square is further subdivided.
- Partitioning continues until a cutoff threshold is reached.

![Quad Tree Example](image-url)
Quad Trees

- Can be represented as a 4-way tree.
- Each node $v$ represents a square $Q_v$
  - If $Q_v \subseteq P$, $v$ is black.
  - If $Q_v \cap P = \emptyset$, $v$ is white.
  - Otherwise $v$ is gray.
  - Gray nodes are further subdivided.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
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| 2 | 3 | Quadrant Numbering

Quad Trees

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Oct Trees

- Oct tree is a similar to quadtrees.
- Each cube is divided into eight octants.
- Useful for many operations, e.g., collision detection, ray tracing.
- Space requirement is still large. Item Sensitive to the position of the object.
- Only approximate representation for nonorthogonal objects.
Boolean Operations on Quad Trees

Object S

Object T

Union (S, T)

Intersection (S, T)
Binary Space Partition (BSP) Trees

$P$: Polyhedron; Normal of each face point to exterior of $P$

- Each interior node $v$ is associated with a plane $\pi_v$ (containing a face of $P$) and convex polytope $Q_v$.
  - $\pi_v^+$: outside halfspace bounded by $\pi_v$.
  - $\pi_v^-$: inside halfspace bounded by $\pi_v$.
- The left child $w$ of $v$ is associated with $Q_v \cap \pi^-$. If $Q_w$ is monochromatic, $w$ is a leaf.
- The right child $x$ of $v$ is associated with $Q_v \cap \pi^+$. If $Q_z$ is monochromatic, $w$ is a leaf.