Relational Model & Algebra

CPS 216
Advanced Database Systems

Announcements (January 13)

- Homework #1 will be assigned on Thursday
- Reading assignment for this week
  - Posted on course Web page
  - Remember to register on H2O and join Duke CPS216
  - Review due on Thursday night

Relational data model

- A database is a collection of relations (or tables)
- Each relation has a list of attributes (or columns)
  - Set-valued attributes not allowed
- Each attribute has a domain (or type)
- Each relation contains a set of tuples (or rows)
  - Duplicates not allowed

= Simplicity is a virtue!
Example

<table>
<thead>
<tr>
<th>Student</th>
<th>Course</th>
<th>Enroll</th>
</tr>
</thead>
<tbody>
<tr>
<td>SID</td>
<td>Name</td>
<td>Age</td>
</tr>
<tr>
<td>142</td>
<td>Bart</td>
<td>10</td>
</tr>
<tr>
<td>123</td>
<td>Milhouse</td>
<td>10</td>
</tr>
<tr>
<td>857</td>
<td>Lisa</td>
<td>8</td>
</tr>
<tr>
<td>456</td>
<td>Ralph</td>
<td>8</td>
</tr>
</tbody>
</table>

Ordering of rows doesn’t matter (even though the output is always in some order)

Example

<table>
<thead>
<tr>
<th>SID</th>
<th>CID</th>
</tr>
</thead>
<tbody>
<tr>
<td>142</td>
<td>CPS216</td>
</tr>
<tr>
<td>142</td>
<td>CPS214</td>
</tr>
<tr>
<td>123</td>
<td>CPS216</td>
</tr>
<tr>
<td>857</td>
<td>CPS216</td>
</tr>
<tr>
<td>857</td>
<td>CPS230</td>
</tr>
<tr>
<td>456</td>
<td>CPS214</td>
</tr>
</tbody>
</table>

Why did Codd call them "relations"?

Schema versus instance

- **Schema** (metadata)
  - Specification of how data is to be structured logically
  - Defined at set-up
  - Rarely changes
- **Instance**
  - Content
  - Changes rapidly, but always conforms to the schema
  - Compare to type and object of type in a programming language

Example

- **Schema**
  - Student (SID integer, name string, age integer, GPA float)
  - Course (CID string, title string)
  - Enroll (SID integer, CID integer)
- **Instance**
  - { (142, Barr, 10, 2.3), (123, Milhouse, 10, 3.1), ... }
  - { (CPS216, Advanced Database Systems), ... }
  - { (142, CPS216), (142, CPS214), ... }
Relational algebra operators

- Core set of operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.

Selection

- Input: a table \( R \)
- Notation: \( \sigma_p(R) \)
  - \( p \) is called a selection condition/predicate
- Purpose: filter rows according to some criteria
- Output: same columns as \( R \), but only rows of \( R \) that satisfy \( p \)

Selection example

- Students with GPA higher than 3.0
  - \( \sigma_{\text{GPA}>3.0} \) (\( \text{Student} \))

<table>
<thead>
<tr>
<th>SID</th>
<th>name</th>
<th>age</th>
<th>GPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>123</td>
<td>Bart</td>
<td>10</td>
<td>2.3</td>
</tr>
<tr>
<td>123</td>
<td>Milhouse</td>
<td>10</td>
<td>3.1</td>
</tr>
<tr>
<td>456</td>
<td>Ralph</td>
<td>8</td>
<td>2.3</td>
</tr>
<tr>
<td>857</td>
<td>Lisa</td>
<td>8</td>
<td>4.3</td>
</tr>
</tbody>
</table>
More on selection

- Selection predicate in general can include any column of R, constants, comparisons such as \( =, \leq \), etc., and Boolean connectives \( \land, \lor, \neg \).
  - Example: straight A students under 18 or over 21
    \[ \sigma_{\text{GPA} \geq 4.0 \land (\text{age} < 18 \lor \text{age} > 21)} (\text{Student}) \]
- But you must be able to evaluate the predicate over a single row
  - Example: student with the highest GPA

Projection

- Input: a table R
- Notation: \( \pi_L (R) \)
  - \( L \) is a list of columns in R
- Purpose: select columns to output
- Output: same rows, but only the columns in \( L \)

Projection example

- ID’s and names of all students

\[ \pi_{\text{SID}, \text{name}} (\text{Student}) \]
More on projection

- Duplicate output rows must be removed
  - Example: student ages

\[ \pi_{age} (\text{Student}) \]

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<th>GPA</th>
</tr>
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<tbody>
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<td>2.3</td>
</tr>
</tbody>
</table>

Cross product

- Input: two tables R and S
- Notation: \( R \times S \)
- Purpose: pairs rows from two tables
- Output: for each row \( r \) in \( R \) and each row \( s \) in \( S \), output a row \( rs \) (concatenation of \( r \) and \( s \))

Cross product example

- \( \text{Student} \times \text{Enroll} \)
A note on column ordering

- The ordering of columns in a table is considered unimportant (as is the ordering of rows)

<table>
<thead>
<tr>
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<th>Name</th>
<th>Age</th>
<th>GPA</th>
<th>CID</th>
</tr>
</thead>
<tbody>
<tr>
<td>142</td>
<td>Bart</td>
<td>10</td>
<td>2.3</td>
<td>CPS216</td>
</tr>
<tr>
<td>142</td>
<td>Bart</td>
<td>10</td>
<td>2.3</td>
<td>CPS214</td>
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<td>CPS214</td>
</tr>
</tbody>
</table>

- That means cross product is commutative, i.e., $R \times S = S \times R$ for any $R$ and $S$

Derived operator: join

- Input: two tables $R$ and $S$
- Notation: $R \bowtie p S$
  - $p$ is called a join condition/predicate
- Purpose: relate rows from two tables according to some criteria
- Output: for each row $r$ in $R$ and each row $s$ in $S$, output a row $rs$ if $r$ and $s$ satisfy $p$
- Shorthand for

Join example

- Info about students, plus CID's of their courses
Derived operator: natural join

- **Input:** two tables $R$ and $S$
- **Notation:** $R \bowtie S$
- **Purpose:** relate rows from two tables, and
  - Enforce equality on all common attributes
  - Eliminate one copy of common attributes
- **Shorthand for $\pi_L (R \bowtie S)$**
  - $L$ is the union of all attributes from $R$ and $S$, with duplicates removed
  - $\bowtie$ equates all attributes common to $R$ and $S$

Natural join example

- $Student \bowtie Enroll = \pi_L (Student \bowtie Enroll)$

Union

- **Input:** two tables $R$ and $S$
- **Notation:** $R \cup S$
  - $R$ and $S$ must have identical schema
- **Output:**
  - Has the same schema as $R$ and $S$
  - Contains all rows in $R$ and all rows in $S$, with duplicates eliminated
Difference

- Input: two tables $R$ and $S$
- Notation: $R - S$
  - $R$ and $S$ must have identical schema
- Output:
  - Has the same schema as $R$ and $S$
  - Contains all rows in $R$ that are not found in $S$

Derived operator: intersection

- Input: two tables $R$ and $S$
- Notation: $R \cap S$
  - $R$ and $S$ must have identical schema
- Output:
  - Has the same schema as $R$ and $S$
  - Contains all rows that are in both $R$ and $S$

Renaming

- Input: a table $R$
- Notation: $\rho_S(R)$, or $\rho_{S(A_1, A_2, \ldots)}(R)$
- Purpose: rename a table and/or its columns
- Output: a renamed table with the same rows as $R$
- Used to
  - Avoid confusion caused by identical column names
  - Create identical columns names for natural joins
Renaming example

- SID’s of students who take at least two courses

Summary of core operators

- Selection: \( \sigma_p (R) \)
- Projection: \( \pi_L (R) \)
- Cross product: \( R \times S \)
- Union: \( R \cup S \)
- Difference: \( R - S \)
- Renaming: \( \rho_{A_1, A_2, \ldots} (R) \)
  - Does not really add to processing power

Summary of derived operators

- Join: \( R \bowtie S \)
- Natural join: \( R \bowtie 1 S \)
- Intersection: \( R \cap S \)
- Many more
  - Semijoin, anti-semijoin, quotient, …
An exercise

- CID's of the courses that Lisa is NOT taking

A trickier exercise

- SID's of students who take exactly one course

Monotone operators

- If some old output rows must be removed
  - Then the operator is non-monotone
- Otherwise the operator is monotone
  - That is, old output rows remain “correct” when more rows are added to the input
  - Formally, $R \subseteq R'$ implies $\text{RelOp}(R) \subseteq \text{RelOp}(R')$
Classification of relational operators

- Selection: $\sigma_p (R)$  Monotone
- Projection: $\pi_L (R)$  Monotone
- Cross product: $R \times S$  Monotone
- Join: $R \bowtie S$  Monotone
- Natural join: $R \bowtie S$  Monotone
- Union: $R \cup S$  Monotone
- Difference: $R - S$  Non-monotone (not w.r.t. $S$)
- Intersection: $R \cap S$  Monotone

Why is “−” needed for “exactly one”?

- Composition of monotone operators produces a monotone query
  - Old output rows remain “correct” when more rows are added to the input
- Exactly-one query is non-monotone
  - Say Nelson is currently taking only CPS216
  - Add another record to Enroll: Nelson takes CPS214 too
  - Nelson is no longer in the answer
- So it must use difference!

Why do we need core operator $X$?

- Difference
  - The only non-monotone operator
- Cross product
- Union

- Selection? Projection?
  - Homework problem 😊
Why is r.a. a good query language?

- Declarative?
  - Yes, compared with older languages like CODASYL
  - But operators are inherently procedural

- Simple
  - A small set of core operators whose semantics are easy to grasp

- Complete?
  - With respect to what?

Relational calculus

- \( \{ s \text{.SID} \mid s \in \text{Enroll} \land \neg \exists e' \in \text{Enroll}: e' \text{.SID} = e \text{.SID} \land e' \text{.CID} \neq e \text{.CID} \} \) or
- \( \{ s \text{.SID} \mid s \in \text{Enroll} \land (\forall e' \in \text{Enroll}: e' \text{.SID} \neq e \text{.SID} \lor e' \text{.CID} \neq e \text{.CID}) \} \)

- Relational algebra = “safe” relational calculus
  - Every query expressible as a safe relational calculus query is also expressible as a relational algebra query
  - And vice versa

- Example of an unsafe relational calculus query
  - \( \{ s \text{.name} \mid \neg (s \in \text{Student}) \} \)
  - Cannot evaluate this query just by looking at the database

Turing machine?

- Relational algebra has no recursion
  - Example of something not expressible in relational algebra: Given relation Parent(parent, child), who are Bart’s ancestors?

- Why not recursion?
  - Optimization becomes undecidable
  - You can always implement it at the application level
  - Recursion is added to SQL nevertheless