Unit 12: Shared-Memory Multiprocessors

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Admin

• Work on Projects
This Unit: Shared Memory Multiprocessors

- Three issues
  - Cache coherence
  - Synchronization
  - Memory consistency

- Two cache coherence approaches
  - “Snooping” (SMPs): < 16 processors
  - “Directory”/Scalable: lots of processors

Thread-Level Parallelism

```
struct acct_t { int bal; };
shared struct acct_t accts[MAX_ACCT];
int id,amt;
if (accts[id].bal >= amt)
{
    accts[id].bal -= amt;
    spew_cash();
}
```

- **Thread-level parallelism (TLP)**
  - Collection of asynchronous tasks: not started and stopped together
  - Data shared loosely, dynamically

- Example: database/web server (each query is a thread)
  - `accts` is **shared**, can’t register allocate even if it were scalar
  - `id` and `amt` are private variables, register allocated to `r1, r2`

- Focus on this
**Shared Memory**

- **Shared memory**
  - Multiple execution contexts sharing a single address space
    - Multiple programs (MIMD)
    - Or more frequently: multiple copies of one program (SPMD)
  - Implicit (automatic) communication via loads and stores
    - Simple software
      - No need for messages, communication happens naturally
        - Maybe too naturally
      - Supports irregular, dynamic communication patterns
        - Both DLP and TLP
  - Complex hardware
    - Must create a uniform view of memory
      - Several aspects to this as we will see

**Shared-Memory Multiprocessors**

- **Provide a shared-memory abstraction**
  - Familiar and efficient for programmers

```
P_1
```

```
P_2
```

```
P_3
```

```
P_4
```

Memory System
Shared-Memory Multiprocessors

- Provide a shared-memory abstraction
  - Familiar and efficient for programmers

![Interconnection Network Diagram]

Paired vs. Separate Processor/Memory?

- **Separate processor/memory**
  - Uniform memory access (UMA): equal latency to all memory
    + Simple software, doesn't matter where you put data
    - Lower peak performance
  - Bus-based UMAs common: symmetric multi-processors (SMP)

- **Paired processor/memory**
  - Non-uniform memory access (NUMA): faster to local memory
    - More complex software: where you put data matters
    + Higher peak performance: assuming proper data placement
Shared vs. Point-to-Point Networks

- **Shared network**: e.g., bus (left) or crossbar (not shown)
  + Low latency
  - Low bandwidth: expensive to scale beyond ~16 processors
  + Shared property simplifies cache coherence protocols (later)
- **Point-to-point network**: e.g., mesh or ring (right)
  - Longer latency: may need multiple “hops” to communicate
  + Higher bandwidth: scales to 1000s of processors
  - Cache coherence protocols are more complex

Organizing Point-To-Point Networks

- **Network topology**: organization of network
  - Tradeoff performance (connectivity, latency, bandwidth) ↔ cost
- Router chips
  - Networks that require separate router chips are **indirect**
  - Networks that use processor/memory/router packages are **direct**
    + Fewer components, "Glueless MP"
  - Distinction blurry in the multicore era
- Point-to-point network examples
  - Indirect tree (left)
  - Direct mesh or ring (right)
Implementation #1: Snooping Bus MP

- Bus-based systems
  - Typically small: 2–8 (maybe 16) processors
  - Typically processors split from memories (UMA)
    - Sometimes **multiple processors on single chip (CMP)**
    - **Symmetric multiprocessors (SMPs)**
    - Common, I use one everyday
  
- Crossbar-based systems similar, but higher B/W and cost

Implementation #2: Scalable MP

- General point-to-point network-based systems
  - Typically processor/memory/router blocks (NUMA)
    - **Glueless MP**: no need for additional “glue” chips
  - Can be arbitrarily large: 1000’s of processors
    - **Massively parallel processors (MPPs)**
  - Increasingly used for small systems
    - Eliminates need for buses, enables point-to-point wires
      - **Coherent Hypertransport (AMD Opteron)**
      - **Intel QuickPath (Core 2)**
Issues for Shared Memory Systems

• Three in particular
  ➢ Cache coherence
  ➢ Synchronization
  ➢ Memory consistency model
  ➢ Not unrelated to each other

• Different solutions for SMPs and MPPs
  ➢ Will discuss SMPs only
  ➢ CMPs? Now like SMPs, but maybe MPPs later

An Example Execution

- Two $100 withdrawals from account #241 at two ATMs
  ➢ Each transaction maps to thread on different processor
  ➢ Track `accts[241].bal` (address is in `r3`)
No-Cache, No-Problem

- Scenario I: processors have no caches
  - No problem

```
Processor 0  
0: addi r1, accts, r3  
1: ld 0(r3), r4  
2: blt r4, r2, 6  
3: sub r4, r2, r4  
4: st r4, 0(r3)  
5: call spew_cash

Processor 1  
0: addi r1, accts, r3  
1: ld 0(r3), r4  
2: blt r4, r2, 6  
3: sub r4, r2, r4  
4: st r4, 0(r3)  
5: call spew_cash
```

Cache Incoherence

- Scenario II: processors have write-back caches
  - Potentially 3 copies of accts[241].bal: memory, p0$, p1$
  - Can get incoherent (inconsistent)

```
Processor 0  
0: addi r1, accts, r3  
1: ld 0(r3), r4  
2: blt r4, r2, 6  
3: sub r4, r2, r4  
4: st r4, 0(r3)  
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Processor 1  
0: addi r1, accts, r3  
1: ld 0(r3), r4  
2: blt r4, r2, 6  
3: sub r4, r2, r4  
4: st r4, 0(r3)  
5: call spew_cash
```
Write-Thru Alone Doesn’t Help

- Scenario II: processors have write-thru caches
  - This time only 2 (different) copies of \texttt{accts[241].bal}
  - No problem? What if another withdrawal happens on processor 0?

<table>
<thead>
<tr>
<th>Processor 0</th>
<th>Processor 1</th>
<th>P0</th>
<th>P1</th>
<th>Mem</th>
</tr>
</thead>
<tbody>
<tr>
<td>0: addi r1, accts, r3</td>
<td>0: addi r1, accts, r3</td>
<td></td>
<td></td>
<td>500</td>
</tr>
<tr>
<td>1: ld 0(r3), r4</td>
<td>1: ld 0(r3), r4</td>
<td></td>
<td></td>
<td>400</td>
</tr>
<tr>
<td>2: blt r4, r2, 6</td>
<td>2: blt r4, r2, 6</td>
<td></td>
<td></td>
<td>400</td>
</tr>
<tr>
<td>3: sub r4, r2, r4</td>
<td>4: st r4, 0(r3)</td>
<td></td>
<td></td>
<td>400</td>
</tr>
<tr>
<td>4: st r4, 0(r3)</td>
<td>5: call spew_cash</td>
<td></td>
<td></td>
<td>300</td>
</tr>
<tr>
<td>5: call spew_cash</td>
<td>500</td>
<td></td>
<td></td>
<td>500</td>
</tr>
</tbody>
</table>

Cloud 220 / ECE 252 (Lebeck): Shared-Memory Multiprocessors

Hardware Cache Coherence

- **Absolute coherence**
  - All copies have same data at all times
  - Hard to implement and slow
  - Not strictly necessary

- **Relative coherence**
  - Temporary incoherence OK (e.g., write-back)
  - As long as all loads get right values
    - i.e., no one looks at incoherent data

- **Coherence controller:**
  - Examines bus traffic (addresses and data)
  - Executes \texttt{coherence protocol}
    - What to do with local copy when you see different things happening on bus

Cloud 220 / ECE 252 (Lebeck): Shared-Memory Multiprocessors
Bus-Based Coherence Protocols

- Bus-based coherence protocols
  - Also called *snooping* or *broadcast*
  - **ALL controllers see ALL transactions IN SAME ORDER**
    - Bus is the *ordering point*
  - Protocol relies on all processors seeing a total order of requests

- Simplest protocol: write-thru cache coherence
  - Two processor-side events
    - **R**: read
    - **W**: write
  - Two bus-side events
    - **BR**: bus-read, read miss on another processor
    - **BW**: bus-write, write thru by another processor

Write-Thru Coherence Protocol

- **VI (valid-invalid) protocol**
  - Two states (per block)
    - **V** (valid): have block
    - **I** (invalid): don't have block
    - Can implement with valid bit

- Protocol diagram (left)
  - Convention: event⇒generated-event
  - Summary
    - If anyone wants to write block
    - Give it up: transition to I state
    - Read miss gets data from memory (as normal)
  - This is an *invalidate protocol*

- Simple, but wastes a lot of bandwidth
  - May be used for L1 D$
Coherence for Writeback caches

- Writeback cache actions
  - Three processor-side events
    - \( R \): read
    - \( W \): write
    - \( WB \): write-back (select block for replacement)
  - Two bus-side events
    - \( BR \): bus-read, read miss on another processor
    - \( BW \): bus-write, write miss on another processor
    - \( CB \): copy-back, send block back to memory or other processor

- Point-to-point network protocols also exist
  - Typical solution is a directory protocol (later)

VI (MI) Coherence Protocol

- **VI (valid-invalid) protocol**: aka MI
  - Two states (per block)
    - \( V \) (valid): have block
      - aka \( M \) (modified) when block written
    - \( I \) (invalid): don’t have block

- Protocol summary
  - If anyone wants to read/write block
    - Give it up: transition to \( I \) state
  - copy-back on replacement or other request
  - Miss gets latest copy (memory or processor)

- This is an **invalidate protocol**

- **Update protocol**: copy data, don’t invalidate
  - Sounds good, but wastes a lot of bandwidth
VI Protocol (Write-Back Cache)

Processor 0
0: addi r1,accts,r3
1: ld 0(r3),r4
2: blt r4,r2,6
3: sub r4,r2,r4
4: st r4,0(r3)
5: call spew_cash

Processor 1
0: addi r1,&accts,r3
1: ld 0(r3),r4
2: blt r4,r2,6
3: sub r4,r2,r4
4: st r4,0(r3)
5: call spew_cash

- **ld** by processor 1 generates a **BR**
  - processor 0 responds by **CB** its dirty copy, transitioning to **I**

VI → MSI: A realistic coherence protocol

- **VI** protocol is inefficient
  - Only one cached copy allowed in entire system
  - Multiple copies can’t exist even if read-only
    - Not a problem in example
    - Big problem in reality

- **MSI** (modified-shared-invalid)
  - Fixes problem: splits "V" state into two states
    - **M** (modified): local dirty copy
    - **S** (shared): local clean copy
  - Allows **either**
    - Multiple read-only copies (S-state) --OR--
    - Single read/write copy (M-state)
MSI Protocol (Write-Back Cache)

Processor 0
0: addi r1,accts,r3
1: ld 0(r3),r4
2: blt r4,r2,6
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5: call spew_cash

- **ld** by processor 1 generates a BR
  - processor 0 responds by CB its dirty copy, transitioning to **S**
- **st** by processor 1 generates a BW
  - processor 0 responds by transitioning to **I**

Cache Coherence and Cache Misses

- Cache parameters interact with coherence misses
  - Larger capacity: more coherence misses
    - But offset by reduction in capacity misses
  - Increased block size: more coherence misses
    - **False sharing**: "sharing" a cache line without sharing data
    - Creates pathological "ping-pong" behavior
    - Careful data placement may help, but is difficult
- Number of processors also affects coherence misses
  - More processors: more coherence misses
Coherence Bandwidth Requirements

- How much address bus bandwidth does snooping need?
  - Well, coherence events generated on...
    - Misses (only in L2, not so bad)
    - Dirty replacements

- Some parameters
  - 2 GHz CPUs, 2 IPC, 33% memory operations,
  - 2% of which miss in the L2, 50% of evictions are dirty
  - \((0.33 \times 0.02) + (0.33 \times 0.02 \times 0.50)\) = 0.01 events/insn
  - 0.01 events/insn * 2 insn/cycle * 2 cycle/ns = 0.04 events/ns
  - Request: 0.04 events/ns * 4 B/event = 0.16 GB/s = 160 MB/s
  - Data Response: 0.04 events/ns * 64 B/event = 2.56 GB/s

- That’s 2.5 GB/s … per processor
  - With 16 processors, that’s 40 GB/s!
  - With 128 processors, that’s 320 GB/s!!
  - Yes, you can use multiple buses... but that hinders global ordering

More Coherence Bandwidth

- Bus bandwidth is not the only problem
- Also **processor snooping bandwidth**
  - Recall: snoop implies matching address against current cache tags
    - Just a tag lookup, not data
  - 0.01 events/insn * 2 insn/cycle = 0.01 events/cycle per processor
  - With 16 processors, each would do 0.16 tag lookups per cycle
    - Add a port to the cache tags ... OK
  - With 128 processors, each would do 1.28 tag lookups per cycle
    - If caches implement inclusion (L1 is strict subset of L2)
      - Additional snooping ports only needed on L2, still bad though

- **Upshot**: bus-based coherence doesn’t scale beyond 8–16
Scalable Cache Coherence

- **Scalable cache coherence**: two part solution

- **Part I**: bus bandwidth
  - Replace non-scalable bandwidth substrate (bus)...
  - ...with scalable bandwidth one (point-to-point network, e.g., mesh)

- **Part II**: processor snooping bandwidth
  - Interesting: most snoops result in no action
    - For loosely shared data, other processors probably
  - Replace non-scalable broadcast protocol (spam everyone)...
  - ...with scalable directory protocol (only spam processors that care)

Directory Coherence Protocols

- Observe: physical address space statically partitioned
  - Can easily determine which memory module holds a given line
    - That memory module sometimes called “home”
  - Can’t easily determine which processors have line in their caches
  - Bus-based protocol: broadcast events to all processors/caches
    - Simple and fast, but non-scalable

- **Directories**: non-broadcast coherence protocol
  - Extend memory to track caching information
  - For each physical cache line whose home this is, track:
    - **Owner**: which processor has a dirty copy (I.e., M state)
    - **Sharers**: which processors have clean copies (I.e., S state)
  - Processor sends coherence event to home directory
    - Home directory only sends events to processors that care
MSI Directory Protocol

- Processor side
  - Directory follows its own protocol (obvious in principle)
- Similar to bus-based MSI
  - Same three states
  - Same five actions (keep BR/BW names)
  - Minus grayed out arcs/actions
    - Bus events that would not trigger action anyway
    - Directory won't bother you unless you need to act

```
I

M

S

W⇒BW
BW⇒WB, WB
R/W⇒WB
R/BR

2 hop miss
3 hop miss

P_0
Dir

P_0
P_1

P0
3 hop miss

P0

P1

Processor 0
0: addi r1,accts,r3
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3: sub r4,r2,r4
4: st r4,0(r3)
5: call spew_cash

P0
P1
Directory

- ld by P1 sends BR to directory
  - Directory sends BR to P0, P0 sends P1 data, does WB, goes to S
- st by P1 sends BW to directory
  - Directory sends BW to P0, P0 goes to I
```

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Directory Flip Side: Latency

- Directory protocols
  - Lower bandwidth consumption → more scalable
  - Longer latencies

- Two read miss situations
  - Unshared block: get data from memory
    - Bus: 2 hops (P0→memory→P0)
    - Directory: 2 hops (P0→memory→P0)
  - Shared or exclusive block: get data from other processor (P1)
    - Assume cache-to-cache transfer optimization
    - Bus: 2 hops (P0→P1→P0)
    - Directory: 3 hops (P0→memory→P1→P0)
      - Common, with many processors high probability someone has it

Directory Flip Side: Complexity

- Latency not only issue for directories
  - Subtle correctness issues as well
  - Stem from unordered nature of underlying inter-connect

- Individual requests to single cache line must appear atomic
  - Bus: all processors see all requests in same order
    - Atomicity automatic
  - Point-to-point network: requests may arrive in different orders
    - Directory has to enforce atomicity explicitly
      - Cannot initiate actions on request B...
      - Until all relevant processors have completed actions on request A
    - Requires directory to collect acks, queue requests, etc.

- Directory protocols
  - Obvious in principle
    - Extremely complicated in practice
One Down, Two To Go

- Coherence only one part of the equation
  - Synchronization
  - Consistency

The Need for Synchronization

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</tbody>
</table>

<table>
<thead>
<tr>
<th>processor 0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

- We're not done, consider the following execution
  - Write-back caches (doesn't matter, though), MSI protocol

- What happened?
  - We got it wrong ... and coherence had nothing to do with it
The Need for Synchronization

- What really happened?
  - Access to accts[241].bal should conceptually be atomic
    - Transactions should not be "interleaved"
    - But that's exactly what happened
    - Same thing can happen on a multiprogrammed uniprocessor!
- Solution: synchronize access to accts[241].bal

Synchronization

- **Synchronization**: second issue for shared memory
  - Regulate access to shared data
  - Software constructs: semaphore, monitor
  - Hardware primitive: lock
    - Operations: acquire(lock) and release(lock)
    - Region between acquire and release is a critical section
    - Must interleave acquire and release
    - Second consecutive acquire will fail (actually it will block)

```
struct acct_t { int bal; };  
shared struct acct_t accts[MAX_ACCT];  
shared int lock;  
int id,amt;  
acquire(lock);

if (accts[id].bal >= amt) {    // critical section
    accts[id].bal -= amt;  
    spew_cash(); }
release(lock);
```

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Working Spinlock: Test-And-Set

- ISA provides an atomic lock acquisition instruction
  - Example: **test-and-set**
    
    \[ \text{t\&s \ r1,0(&lock)} \]
    
    - Atomically executes
      \[ \text{ld \ r1,0(&lock)} \]
      \[ \text{st \ 1,0(&lock)} \]
    - If lock was initially free (0), acquires it (sets it to 1)
    - If lock was initially busy (1), doesn’t change it
  - New acquire sequence
    \[ \text{A0: t\&s \ r1,0(&lock)} \]
    \[ \text{A1: bnez \ r1,A0} \]
  - More general atomic mechanisms
    - **swap, exchange, fetch-and-add, compare-and-swap**

Test-and-Set Lock Correctness

<table>
<thead>
<tr>
<th>Processor 0</th>
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</tr>
</thead>
<tbody>
<tr>
<td>A0: t&amp;s \ r1,0(&amp;lock)</td>
<td>A0: t&amp;s \ r1,0(&amp;lock)</td>
</tr>
<tr>
<td>A1: bnez \ r1,#A0</td>
<td>A0: t&amp;s \ r1,0(&amp;lock)</td>
</tr>
<tr>
<td>CRITICAL SECTION</td>
<td>A1: bnez \ r1,#A0</td>
</tr>
<tr>
<td>A0: t&amp;s \ r1,0(&amp;lock)</td>
<td>A0: t&amp;s \ r1,0(&amp;lock)</td>
</tr>
<tr>
<td>A1: bnez \ r1,#A0</td>
<td>A0: t&amp;s \ r1,0(&amp;lock)</td>
</tr>
</tbody>
</table>

+ Test-and-set lock actually works
  - Processor 1 keeps spinning
Memory Consistency

- **Memory coherence**
  - Creates globally uniform (consistent) view...
  - Of a single memory location (in other words: cache line)
    - Not enough
      - Cache lines A and B can be individually consistent...
      - But inconsistent with respect to each other

- **Memory consistency**
  - Creates globally uniform (consistent) view...
  - Of all memory locations relative to each other

- Who cares? Programmers
  - Globally inconsistent memory creates mystifying behavior

Coherence vs. Consistency

- Intuition says: P1 prints A=1
- Coherence says?
- Absolutely nothing!
  - P1 can see P0’s write of flag before write of A!!! How?
    - Maybe coherence event of A is delayed somewhere in network
    - Maybe P0 has a coalescing write buffer that reorders writes
- Imagine trying to figure out why this code sometimes “works” and sometimes doesn’t
- **Real systems** act in this strange manner

```
A=flag=0;
Processor 0
A=1;
flag=1;
Processor 1
while (!flag); // spin
print A;
```
Sequential Consistency (SC)

- Sequential consistency (SC)
  - Formal definition of memory view programmers expect
  - Processors see their own loads and stores in program order
    + Provided naturally, even with out-of-order execution
  - But also: processors see others’ loads and stores in program order
  - And finally: all processors see same global load/store ordering
    – Last two conditions not naturally enforced by coherence
- Lamport definition: multiprocessor ordering...
  - Corresponds to some sequential interleaving of uniprocessor orders
  - I.e., indistinguishable from multi-programmed uni-processor

Enforcing SC

- What does it take to enforce SC?
  - Definition: all loads/stores globally ordered
  - Translation: coherence events of all loads/stores globally ordered
- When do coherence events happen naturally?
  - On cache access
  - For stores: retirement → in-order → good
    • No write buffer? Yikes, but OK with write-back D$
  - For loads: execution → out-of-order → bad
    – No out-of-order execution? Double yikes
- Is it true that multi-processors cannot be out-of-order?
  – No, but it makes OoO a little trickier
  - Treat out-of-order loads and stores as speculative
  - Treat certain coherence events as mispeculations
    • E.g., a BW request to block with speculative load pending
Multiprocessors Are Here To Stay

- Moore’s law is making the multiprocessor a commodity part
  - >1B transistors on a chip, what to do with all of them?
  - Not enough ILP to justify a huge uniprocessor
  - Really big caches? $t_{\text{hit}}$ increases, diminishing $\%_{\text{miss}}$ returns

- Chip multiprocessors (CMPs)
  - Multiple full processors on a single chip
  - Example: IBM POWER4: two 1GHz processors, 1MB L2, L3 tags
  - Example: Sun Niagara: 8 4-way FGMT cores, 1.2GHz, 3MB L2

- Multiprocessors a huge part of computer architecture
  - Another entire course on multiprocessor architecture

Multiprocessing & Power Consumption

- Multiprocessing can be very power efficient

- Recall: dynamic voltage and frequency scaling
  - Performance vs power is NOT linear
  - Example: Intel’s Xscale
    - 1 GHz → 200 MHz reduces energy used by 30x

- Impact of parallel execution
  - What if we used 5 Xscales at 200Mhz?
  - Similar performance as a 1Ghz Xscale, but $1/6$th the energy
    - 5 cores * 1/30th = 1/6th

- Assumes parallel speedup (a difficult task)
  - Remember Ahmdal’s law
**Shared Memory Summary**

- **Three aspects to global memory space illusion**
  - **Coherence**: consistent view of individual cache lines
    - Implementation? SMP: snooping, MPP: directories
  - **Synchronization**: regulated access to shared data
    - Key feature: atomic lock acquisition operation (e.g., `t&s`)
  - **Consistency**: consistent global view of all memory locations
    - Programmers intuitively expect sequential consistency (SC)

- **How do we implement this**
  - Correctly
  - Cost-Effectively
  - **TAKE CompSci 221/ECE 259!!**