**Assignment 4**  

**Course: CPS530**

**Due Date: Nov 6, 2012**

**Problem 1:** For a graph $G = (V, E)$ with maximum clique size $k$, suppose there is a polynomial-time algorithm for finding a clique with size at least $k/2$. Based on this algorithm, describe a new algorithm (using this algorithm as a subroutine) to find a clique in $G$ of size at least $k/\sqrt{2}$ in polynomial time. (Hint: Construct the new graph $G^2$ that has $|V|^2$ many nodes.)

**Problem 2:** A function $f : V \rightarrow \{1, 2, \ldots, k\}$ is called a $k$-coloring of the graph $G = (V, E)$ if for any edge $(u, v) \in E$, we have $f(u) \neq f(v)$. A graph is $k$-colorable if there exists a $k$-coloring of that graph.

(i) Given a graph $G$ with maximum degree $d$ (a vertex has at most $d$ adjacent edges), describe a polynomial algorithm to find a $d + 1$-coloring of that graph.

(ii) Given a graph $G$, describe a polynomial algorithm to decide whether this graph is 2-colorable, and compute a 2-coloring of that graph if it’s 2-colorable.

(iii) Given a 3-colorable graph $G$, describe a polynomial algorithm to compute an $O(\sqrt{n})$-coloring of that graph. (Hint: Combine (i) and (ii).)

**Problem 3:** Assume $S$ is a set of $n$ distinct real numbers. We say that a number $x$ is an $\varepsilon$-approximate median of $S$ if at least $(\frac{1}{2} - \varepsilon)n$ numbers in $S$ are less than $x$, and at least $(\frac{1}{2} - \varepsilon)n$ numbers in $S$ are greater than $x$. Consider an algorithm that works as follows. Select a subset $S' \subseteq S$ uniformly at random, compute the median of $S'$, and return this as an approximate median of $S$. Show that there is a constant $c$, independent of $n$, so that if you apply this algorithm with a sample $S'$ of size $c$, then with probability at least .99, the number returned will be a (.05)-approximate median of $S$. (You may consider either the version of the algorithm that constructs $S'$ by sampling with replacement, or one without).

**Problem 4:** Suppose there are $n$ items of weight $w_1, \ldots, w_n$ that need to be packed in containers, each of which can contain at most $K$ units of weight. The goal is to pack these items in the minimum number of containers. Let $m^*$ be the minimum number of containers needed to pack these items.

Consider the following greedy algorithm: Start with an empty container and begin filling items 1, 2, 3 into it until you get an item that would overflow the weight limit of the container. Declare this container “full” and start a new container and repeat the same process. Once a container is declared “full,” no other item is put in it.

(i) Show that the greedy algorithm uses at most $2m^*$ containers.

(ii) Show there is a sequence of $n$ items on which the algorithm uses at least $2m^* - 2$ containers.