CPS234 COMPUTATIONAL GEOMETRY

Homework # 3

Due date: March 2, 2004, Tuesday (beginning of the class)

Your solutions should be concise, but complete, and typed (or handwritten clearly). Feel free to consult textbooks, journal and conference papers and also each other, but write the solutions yourself and cite your sources. Answer **only four** of the following five questions.

1. **Vertical order in monotone subdivision** [25 pts]. Given two subsets $r, s \subseteq \mathbb{R}^2$, we say that r is *below* s, denoted by $r \prec s$ if for any $(p_x, p_y) \in r$ and $(q_x, q_y) \in s$, we have $p_y \leq q_y$ if $p_x = q_x$. Show that the relation $\prec$ is acyclic if applied to the regions of a monotone subdivision of $\mathbb{R}^2$. Construct an example to show that this is not the case for a monotone cell decomposition of $\mathbb{R}^3$.

2. **Ray shooting in simple polygon** [25 pts]. Given an n-vertex simple polygon P and an edge e of P, show how to construct a data structure to answer the following queries in $O(\log n)$ time and $O(n)$ space. Given a ray r whose origin lies on e and which is directed into the interior of P, find the first edge of P that r hits.

3. **Planar separator theorem** [25 pts]. The theorem states that every planar graph of n nodes has a set of $O(\sqrt{n})$ nodes, the removal of which will partition the graph into two or more pieces each with less than $2n/3$ nodes. Show that given a planar subdivision, the theorem can be exploited to construct a point location data-structure of $O(n)$ size and provides $O(\log^2 n)$ query time.

4. **Dynamic planar point location** [25 pts]. Given a planar subdivision with n vertices, use interval tree and priority search tree to construct a data-structure of $O(n)$ size such that a point location query can be answered in $O(\log^2 n)$ time. This structure can be made dynamic to allow inserting/deleting an edge in $O(\log n)$ time. Explain briefly your ideas on how to achieve it.

5. **Voronoi diagram in L_∞ metric** [25 pts]. The L_∞ metric measures distance between two points $p, q \in \mathbb{R}^2$ is as:
$$d_\infty(p, q) = max(|p_x - q_x|, |p_y - q_y|).$$
Describe the set of points that are equidistant from p and q in the L_∞ distance. Pay special attention to degenerate cases. Argue that the Voronoi diagram for n points using L_∞ metric has $O(n)$ edges and vertices (assume there is no degeneracy).