CPS234 COMPUTATIONAL GEOMETRY

Homework # 7

This set of homework questions is for practice only and solutions do not have to be turned in.

1. **Möbius strip** [25 pts] Recall that a Möbius strip is obtained by gluing two opposite sides of a rectangular piece of paper, after giving it a twist. The result is a non-orientable 2-manifold with a single circle as its boundary.

   (i) What 2-manifold (without boundary) do you get when you attach a disk by gluing its boundary circle to the boundary circle of the Möbius strip?

   (ii) What 2-manifold (without boundary) do you get when you glue antipodal points on the boundary circle of the Möbius strip?

2. **Critical points** [25 pts] Let $f : \mathbb{M}^3 \to \mathbb{R}$ be a Morse function on a 3-manifold. There are four types of critical points distinguished by their indices. What are these types? Draw the lower link of each type.

3. **Induced subcomplexes** [25 pts] Let K be a triangulation of a 2-manifold and $f : K \to \mathbb{R}$ a piecewise linear map defined by its values at the vertices. The subcomplex *induced* by a subset U of the vertices in K consists of all simplices spanned by vertices in U.

   (i) Draw an instructive example that shows a level set, $f^{-1}(t)$, the subcomplex induced by vertices $f(u) \leq t$, and the subcomplex induced by vertices $f(u) > t$.

   (ii) Prove that the first subcomplex in (i) is homotopy equivalent to K_t, the set of points with $f(x) \leq t$. [Hint: use a deformation retraction from K_t to the subcomplex.]

4. **Iterated stars** [25 pts] Let K be a simplicial complex and $L \subseteq K$ a subset of the simplices. The *star* of L is the set of simplices that contain at least one simplex in L as a face. Define $L^0 = L$ and $L^k = \text{St}\, L^{k-1}$ for $k \geq 1$.

   (i) Draw instructive examples in which L consists of one, two, and three vertices.

   (ii) Is it true that if K triangulates the 2-sphere and L consists of a single vertex, then L^k is a topological disk for each $k \geq 1$?

5. **2-manifold with boundary** [25pts] Recall that two connected 2-manifolds with boundary are homeomorphic iff they are both orientable or both non-orientable, they have the same genus, and they have the same number of boundary cycles. Which non-homeomorphic 2-manifolds with boundary cannot be distinguished by their Betti numbers?