Due date: September 9, Monday, the beginning of the class.
Credits: 10 full + 4 bonus

1. (two credits) Show that the following procedure returns twice the signed area of a given triangle \( \triangle(abc) \).

\[
\text{AREA } (a, b, c \in \mathbb{R}^2) = (c_y - a_y)(b_x - a_x) - (b_y - a_y)(c_x - a_x);
\]

2. (two credits) Prove or disprove: The dual graph of the triangulation of a monotone polygon is always a chain, that is any node in this graph has degree at most two.

3. (four credits) Let \( K \) be a triangulation of a set of \( n \) points in the plane. Let \( \ell \) be a line that avoids all points. Prove that \( \ell \) intersects at most \( 2n - 4 \) edges of \( K \) and that this upper bound is tight for every \( n \geq 3 \).

4. (four credits) A \( k \)-coloring of a graph \( G(V, E) \) is a function \( \gamma : V \rightarrow \{1, 2, \ldots, k\} \) such that \( \gamma(u) \neq \gamma(v) \) if \( (u, v) \in E \). Prove that a planar triangulation has a 6-coloring.

5. (two credits) An ear is a triangle bounded by a diagonal and two polygon edges. Prove that every triangulation of an \( n \)-gon has to have at least one ear, provided \( n \geq 4 \).