1. (four credits) The *aspect ratio* of a simplex is the ratio of its circumradius to inradius. Show that this quality measure is equivalent to smallest angle measure, i.e.,

(a) a lower bound on the smallest angle implies an upper bound on the aspect ratio;
(b) an upper bound on the aspect ratio implies a lower bound on the smallest angle.

2. (three credits) The body centered cube (BCC) lattice is the set of points \((i, j, k)\), \((i + 1/2, j + 1/2, k + 1/2)\) for \(i, j, k \in \mathbb{Z}\). Delaunay triangulation of BCC lattice consists of congruent copies of a single tetrahedron. Determine all the metric properties of this tetrahedron: volume, areas of triangles, length of edges, face angles, dihedral angles, and solid angles.

3. (four credits) Let \(I^3 = [0, 1] \times [0, 1] \times [0, 1]\) be the unit cube in \(\mathbb{R}^3\) and consider a triangulation \(K\) of \(I^3\) whose only vertices are the 8 corner points of the cube.

(a) Show that every such \(K\) has at most 6 tetrahedra.
(b) Show that every such \(K\) has at least 5 tetrahedra.
(c) Two triangulations \(K_1\) and \(K_2\) are *isomorphic* if \(\exists\) a bijection \(\beta : Vertices(K_1) \rightarrow Vertices(K_2)\) such that \(ConvHull(T) \in K_1\) iff \(ConvHull(\beta(T)) \in K_2\). Enumerate all pairwise non-isomorphic triangulations of unit cube (with no Steiner points).