This is a full length practice final exam. If you want to take it at exam pace, give yourself 75 minutes to take the entire test. Just like the real exam, each question has a point value. There are 75 points in the exam, so that you can pace yourself to average 1 point per minute (some parts will be faster, some slower).

Questions:

1. Types (25 points)
2. Frame Layout (10 points)
3. Translation to IR (10 points)
4. Register Allocation (15 points)
5. Dataflow Analysis/Optimization (15 points)
6. Domination (10 points)
7. Garbage Collection (15 points)

This is the solution set to the practice exam. The solutions appear in blue boxes.
Question 1: Types [25 pts]

1. Show the typing derivation for the Tiger statement $x := f(r.a) + 3$.
   You may assume that your initial environment ($\Gamma_0$) has the following mappings (in addition to the base Tiger environment):
   $\Gamma_0(x) = \text{int}$
   $\Gamma_0(a) = \text{int}$
   $\Gamma_0(r) = \text{Record(a:string, b:int)}$
   $\Gamma_0(f) = \text{string} \rightarrow \text{int}$

Answer:

\[
\begin{array}{c}
\Gamma_0 \vdash f : \text{string} \rightarrow \text{int} \\
\Gamma_0 \vdash r : \text{Record(a:string,\ldots)} \\
\Gamma_0 \vdash r.a : \text{string} \\
\end{array}
\]

\[
\begin{array}{cc}
\Gamma_0 \vdash f(r.a) : \text{int} & \Gamma_0 \vdash 3 : \text{int} \\
\end{array}
\]

\[
\begin{array}{c}
\Gamma_0 \vdash x : \text{int} \\
\Gamma_0 \vdash f(r.a) + 3 : \text{int} \\
\end{array}
\]

\[
\Gamma_0 \vdash x := f(r.a) + 3 : \text{unit}
\]
2. Fill in the correct premises for the sub-typing rule for function types. Then briefly explain why it is correct:

\[
\begin{align*}
T_1 & \sqsubseteq S_1 & S_2 & \sqsubseteq T_2 \\
S_1 \to S_2 & \sqsubseteq T_1 \to T_2
\end{align*}
\]

**Answer:**
To understand this rule, suppose we have a function \( f \) of type \( S_1 \to S_2 \), and want to use it as a function of type \( T_1 \to T_2 \) (e.g., pass it as a parameter, assign it to a variable etc.). We need to be sure that \( f \) can handle any \( T_1 \) it may be passed as its argument, and that its calling context can handle any \( T_2 \) that \( f \) may return as its result.

The first constraint requires *contra-variance* in the argument type. This guarantees that whatever we actually pass into the function (e.g., \( T_1 \)) will always be acceptable relative to what the function expects (e.g., a sub-type of \( S_1 \)).

The second constraint requires *co-variance* in the return type. This guarantees that whatever the function returns (e.g., \( S_2 \)) will be acceptable to what its calling context expects (e.g., a sub-type of \( T_2 \)).

3. Infer the type of the following ML function (show your work):

```ml
fun f (w,x) = case x of
  [] => []
| y::z => w(y)::f(w,z)
```

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Answer:
First, assign type variables where appropriate:
fun f (w:'a,x:'b):'c = case x of
  [] => []
  | (y:'d)::(z:'e) => w(y)::f(w,z)

Then start unifying:

<table>
<thead>
<tr>
<th>Must Match</th>
<th>So unify</th>
<th>Resulting in</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>[]</td>
<td>'b</td>
</tr>
<tr>
<td>(y,z)</td>
<td>cons’s arg</td>
<td>('d * 'e)</td>
</tr>
<tr>
<td>x</td>
<td>cons's rslt</td>
<td>'f list</td>
</tr>
<tr>
<td>w</td>
<td>fn taking y</td>
<td>'a</td>
</tr>
<tr>
<td>(w,z)</td>
<td>f's arg</td>
<td>('h → 'i) * 'h list</td>
</tr>
<tr>
<td>(w(x),f(w,z))</td>
<td>cons’s arg</td>
<td>'i * 'c</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; case rslt</td>
<td>fn rslt ty</td>
<td>'k list</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; case rslt</td>
<td>fn rslt ty</td>
<td>'j list</td>
</tr>
</tbody>
</table>

Replace types:
fun f (w:'h → 'j ,x:'h list):'j list = case x of
  [] => []
  | (y:'h)::(z:'h list) => w(y)::f(w,z)

4. What goes wrong if you attempt type inference on \texttt{fun f(x) = f}?

Answer:
When you attempt to run type-inference on \texttt{fun f(x) = f}, you try to unify 'a with ('a → 'b), which fails the occurs check.
Question 2: Frame Layout [10 pts]

• Explain the concept of a static link. Why is it needed?

Answer:
The static link is the frame pointer of the statically enclosing function's stack frame. This frame may dynamically be one or many frames out. The static link is required to allow access to variables declared in outer-functions, which reside in that function's frame.

• Explain the difference between the stack pointer and the frame pointer. One of them can be omitted in certain circumstances. Identify which one, and explain when it is not needed, and what benefits are obtained from omitting it.

Answer:
The FP points at the “start” of the current functions stack frame. Variables are always a fixed offset from the FP. The SP, on the other hand points at the end of the stack. In the presence of dynamic stack allocation (alloca, variable sized arrays, etc), the offset of a variable from the SP may change depending on the size of the dynamically allocated structures. The FP could be omitted when no dynamic allocation is performed. Omitting the FP saves a few instructions at function entry and exit, improving the speed of the program.
Question 3: Translation to IR [10 pts]
Translate the following bits of Tiger into IR (you can either draw the IR tree or write it out as SML constructors). For each case you should assume the following variable locations (all InFrame variables are in your own frame. You can refer to the frame pointer as simply FP). Note: you do not need to include bounds checks for array accesses:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>InReg t1</td>
</tr>
<tr>
<td>y</td>
<td>InReg t2</td>
</tr>
<tr>
<td>z</td>
<td>InFrame -4</td>
</tr>
<tr>
<td>a</td>
<td>InReg t3</td>
</tr>
</tbody>
</table>

- y := 1 + x
  
  **Answer:**
  
  MOVE(TEMP t2 , BINOP(PLUS,CONST 1 ,TEMP t1))

- a[y] := f(z)
  
  **Answer:**
  
  MOVE(MEM(BINOP(PLUS, 
  TEMP t3, 
  BINOP(TIMES, TEMP t2, CONST 4))), 
  CALL(LABEL("f"),[MEM(BINOP(PLUS,FP,CONST -4))])))
Question 4: Register Allocation [15 pts]
Perform register allocation for a 3 register machine on the following interference graph (dashed lines indicate move relationships, solid lines indicate interference). You should coalesce moves whenever it is safe to do so according to either heuristic we learned. Show your work (you do not need to redraw the graph for each step, but you should list the order in which you simplify/coalesce/freeze nodes)

Answer:
Simplify b
Coalesce p and q
Simplify pq
Freeze x and c
Simplify x, y, a, z, c
Registers: c: r1, z: r2, a: r3, y: r1, x: r2, pq: r2, b r3
Question 5: Dataflow Analysis/Optimization [15 pts]

1. Using the above program fragment, which definitions reach the following uses (you can identify them by their instruction number):
   - The use of a in instruction 4. 1,8
   - The use of a in instruction 8. 1,8,13
• The use of \( b \) in instruction 6. 2,11

2. For the same program fragment, indicate whether each of the following expressions is “very busy” (write Y or N) after each block (after the last instruction in that block, numbered down the left side of the table):

<table>
<thead>
<tr>
<th></th>
<th>a + 1</th>
<th>m - 1</th>
<th>a + b</th>
<th>b * 47</th>
<th>x + y</th>
<th>b + 1</th>
<th>arr[b]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>7</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>14</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>15</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

3. For each of the following, indicate the appropriate data flow analysis (you can just write one of RD, LV, VBE, or AE on each line):

• Common sub-expression elimination AE
• Forward flow/union RD
• Def/Use Web Formation RD
• Backwards flow/intersection VBE
Question 6: Domination [10 pts]
Consider the following control flow graph:

- Label each node in the graph with its dominator set. Done in place above
- Identify the loops in the graph by their backedges.

**Answer:**
3 → 3 is one loop. The other is 8 → 4. Note that 4 → 1 is NOT a loop (see next question).
• This control flow graph has a part that looks like a loop to a naive definition, but is not a loop for the definition required for many optimizations. Identify this false loop and briefly explain why it is not a loop.

**Answer:**
4 makes a backwards edge to 1, which meets a naive definition of a loop (e.g., a cycle). However, 1 does not dominate 4 ($0 \rightarrow 2 \rightarrow 3 \rightarrow 4$ is a possible path), so this is not a true loop.

• Draw the immediate dominator tree for this graph.

**Answer:**

![Immediate Dominator Tree](image)
Question 7: Garbage Collection [15 pts]

• List three advantages of garbage collection. You may list advantages particular to one specific GC algorithm, but please specify which algorithm if you do so.

   Answer:
   Any 3 of these (plus possibly others): fast allocation (S&C), improved locality (S&C), avoid double frees, avoid free-then-use errors, avoid memory leaks, reduce development/debugging time/cost, ...

• Briefly describe the performance/space tradeoffs between the three algorithms we discussed: reference counting, mark and sweep, and stop and copy.

   Answer:
   Reference counting has very high performance overhead: it requires stores every time pointers are manipulated, typically resulting in 20–30 word per object to hold the count. Mark and sweep requires a DFS (time proportional to the live objects) and examination of all objects (time proportional to the total heap size). When the heap size is large relative to the number of live objects, this amortized cost is acceptable. The direct space overhead is one word per object (to hold the marking state). Stop and copy requires time proportional to the number of live objects. It has large space overhead—half the heap must be unused to copy into. There may be an additional overhead of one word per object to hold forwarding pointers, depending on the object layout.

• The garbage collector needs to know which fields in an object are pointers, and which are not. Briefly describe two ways it might do this.

   Answer:
   Any two of these three: The compiler could pass down type information to tell it. It could be conservative: anything that looks like a pointer might be (but then no moving objects). It could use tagging: steal one bit from each word to indicate “pointer or not”.

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