

## Sixth Homework Assignment

**Question 1.** ( $20 = 5 + 5 + 5 + 5$  points). Choose ten of your friends, and make a graph where the edges represent two friends being Facebook friends. (Do not include yourself in the graph). Order your friends alphabetically, and label the vertices  $v_1, v_2, \dots, v_{10}$  respectively. This will be most interesting if all of your friends know each other. Now, answer the following questions about the graph that you drew.

- (a) What is the size of the largest clique?
- (b) Find the shortest and longest paths from  $v_1$  to  $v_{10}$ .
- (c) Which vertex has the highest degree?
- (d) Use Prim's algorithm to find the spanning tree, and draw that tree.

**Question 2.** (20 points). (Problem 6.1-9 in our textbook). A graph with no cycles is called a *forest*. Show that if a forest has  $n$  vertices,  $m$  edges, and  $c$  connected components, then  $n = m + c$ .

**Question 3.** (20 points). Call a simple graph with  $n \geq 3$  vertices an *Ore graph* if every pair of non-adjacent vertices has a combined degree of at least  $n$ . Is it true that every Ore graph is Hamiltonian? Justify your answer.

**Question 4.** ( $20 = 10 + 10$  points). (Problems 6.4-12 and 13 in our textbook). Prove or give a counterexample:

- (a) Every tree is a bipartite graph (allowing for the number of vertices in one set or the other to be zero).
- (b) A bipartite graph has no odd cycles.

**Question 5.** (20 points). (Problems 6.5-13 in our textbook). Show that in a simple planar graph with no triangles,  $m \leq 2n - 4$ .