CPS 130: Homework 6

Graph Algorithms

(CLRS: Chapter 22)

Date on which distributed: Thursday, October 18, 2001

Date on which due: Thursday, October 25, 2001

Note: Zero credit will be given for homeworks submitted late.

1. [Exc 22.1-4 CLRS]
Given an adjacency-list representation of a multigraph $G = (V, E)$, describe an $O(V + E)$-time algorithm to compute the adjacency-list representation of the "equivalent" undirected graph $G' = (V, E')$, where E' consists of the edges in E with all multiple edges between two vertices replaced by a single edge and with all self-loops removed. Explain why the algorithm is $O(V + E)$.

2. [Exc 22.2-8 CLRS]
Let $G = (V, E)$ be a connected, undirected graph. Give an $O(V + E)$-time algorithm to compute a path in G that traverses each edge in E exactly once in each direction. Explain why the algorithm is $O(V + E)$.

(Hint: Consider a graph $G' = (V, E')$, where each undirected edge e in E is replaced with a pair of directed edges e', e'' in E' joining the end-vertices of e, one directed edge going in one direction, and the other directed edge going in the other. Looking at the in-degrees and out-degrees at each node of G' may help find the required path in G.)

3. [Exc 22.3-6 CLRS]
Show how you would represent a Depth-first search using a *stack*. Justify your answer. Illustrate your algorithm on a small graph, say one with 4 or 5 vertices.

4. [Exc 22.4-3 CLRS]
Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(V)$ time, independent of $|E|$. Justify your answer.