CPS 130: Homework 7

More Graphs: Spanning Trees, Shortest Paths

(CLRS: Chapter 23.2, 24.3, 25.2)

Date on which distributed: Thursday, November 01, 2001

Date on which due: Tuesday, November 13, 2001

Note: Zero credit will be given for homeworks submitted late.

1. [Exc 23.2-2 CLRS]
Suppose that the graph $G = (V, E)$ is represented as an adjacency matrix. Give a simple implementation of Prim's algorithm for this case that runs in $O(V^2)$ time.

2. [Exc 23.2-4 CLRS]
We know that Kruskal's algorithm runs in $O(E \lg V)$ time on a graph $G = (V, E)$. Suppose that all edge weights in G are integers in the range from 1 to $|V|$. How fast can you make Kruskal's algorithm on G? What if the edge weights are integers in the range from 1 to W for some constant W? (You can assume W is an integer too.)

3. [Exc 24.3-4 CLRS]
We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$ which is a real number in the range $0 \leq r(u, v) \leq 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices. (Hint: The probability of the *success* of any path from u to v is the *product* of the probabilities of success of the individual edges that make up this path. You may want to try to convert the problem to a Dijkstra-like situation. If this is your approach, remember that Dijkstra's algorith finds the *shortest* path between any two given vertices, and that weights of paths are computed as *sums*, not *products* of weights of edges along them.)

4. [Exc 25.2-6 CLRS]
How can the output of the Floyd-Warshall algorithm be used to detect the presence of a negative-weight cycle?