CPS 230 **Prof. Jeff Vitter**
Design and Analysis of Algorithms **Fall 2001**

Homework 1 (due before class, Monday, September 10, 2001)

1 Growth of Functions [Chapter 2 and 3 in CLRS]

1. [**CLRS 3.1-1**] Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ-notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

2. [**CLRS 3.3 (a)**] Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots, g_{30}$ of functions satisfying $g_1 = \Omega(g_2)$, $g_2 = \Omega(g_3), \ldots, g_{29} = \Omega(g_{30})$. Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$:
$\lg(\lg^* n)$, $2^{\lg^* n}$, $\sqrt{2}^{\lg n}$, n^2, $n!$,
$(\lg n)!$,$(\frac{3}{2})^n$, n^3, $\lg^2 n$, $\lg(n!)$,
2^{2^n}, $n^{1/\lg n}$, $\ln \ln n$, $\lg^* n$, $n2^n$,
$n^{\lg \lg n}$, $\ln n$, 1, $2^{\lg n}$, $(\lg n)^{\lg n}$,
e^n, $4^{\lg n}$, $(n+1)!$, $\sqrt{\lg n}$, $\lg^*(\lg n)$,
$2^{\sqrt{2 \lg n}}$, n, 2^n, $n \lg n$, $2^{2^{n+1}}$

2 Summations [Appendix A of CLRS]

3. Compute $\sum_{k=1}^{\log_3 n} k/3^k$, $\sum_{k=1}^{n} k/3^k$, and $\sum_{k=1}^{\infty} k/3^k$. Give the exact answers. What is the leading term in each?

4. [**CLRS A-1**] Give a Θ bound on the summation $\sum_{k=1}^{n} k^r \lg^s k$. Assume $r, s \geq 0$ are constants.

5. Prove by induction that $\sum_{i=1}^{n} i(i!) = (n+1)! - 1$.

3 Strassen's Algorithm [Chapter 28 of CLRS]

6. [**CLRS 28.2-3**] Assume we do a divide-and-conquer of each $n \times n$ matrix that we want to multiply, breaking it into nine $\frac{n}{3} \times \frac{n}{3}$ submatrices rather than into four $\frac{n}{2} \times \frac{n}{2}$ submatrices. Let n be a power of 3. What is the largest k such that if you can multiply 3×3 matrices using k multiplications (not assuming commutativity of multiplication), then you can multiply $n \times n$ matrices in time $o(n^{\lg 7})$? What would the running time of this algorithm be?