CPS 230 **Prof. Jeff Vitter**

Design and Analysis of Algorithms **Fall 2001**

Homework 2 (due before class, Monday, September 24, 2001

1 Recurrences, Master Method, Sorting [Chapter 4 in CLRS]

1. [**CLRS 4.2-5**] Solve the recurrence $T(n) = T(\alpha n) + T((1-\alpha)n) + n$, where $0 < \alpha < 1$ constant. (Hint: Use a recursion tree. Use the parsetree.sty package to LaTeX the tree.)

2. Give asymptotic upper and lower bounds using Θ notation for $T(n)$ at those points where $T(n)$ is defined, for the following recurrences. Assume $T(n)$ is constant for $n \leq 20$. Make your bounds as tight as possible, and justify your answers.

(a) $T(n) = 2T(n/3) + n^3$

(b) $T(n) = 3T(n/4) + \sqrt{n}$

(c) $T(n) = 9T(n/4) + n^2$

(d) $T(n) = T(n-6) + n$

(e) $T(n) = T(\sqrt{n}) + 1$

(f) $T(n) = 2T(n/2) + n \lg^2 n$

(g) $T(n) = 3T(n/3 + 5) + n/2$

(h) $T(n) = 2T(n/2) + n/\lg n$

(i) $T(n) = T(n-2) + 1/n$

(j) $T(n) = \sqrt{n}T(\sqrt{n}) + n$

2 Quicksort, Median, Matrices [Chapter 7, 9, 28 in CLRS]

3. [**CLRS 9.3-1**] In the algorithm `Select`, the input elements are divided into groups of 5. Will the algorithm work in linear time if they are divided into groups of 7? How about groups of 3?

4. [**CLRS 9-2**] For n distinct elements $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2, \ldots, w_n$ such that $\sum_{i=0}^{n} w_i = 1$, the weighted (lower) median is the element x_k satisfying

$$\sum_{x_i < x_k} w_i < \frac{1}{2} \text{ and } \sum_{x_i > x_k} w_i \leq \frac{1}{2}$$

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(a) Argue that the median of $x_1, x_2, \ldots, x_n$ is the weighted median of the x_i with weights $w_i = 1/n$ for $i = 1, 2, \ldots, n$.

(b) Show how to compute the weighted median of n elements in $O(n \lg n)$ worst-case time using sorting.

(c) Show how to compute the weighted median in $\Theta(n)$ worst-case time using a linear-time median algorithm.

The post office location problem is defined as follows. We are given n points $p_1, p_2, \ldots, p_n$ with associated weights $w_1, w_2, \ldots, w_n$. We wish to find a point p (not necessarily one of the input points) that minimizes the sum $\sum_{i=1}^{n} w_i d(p, p_i)$, where $d(a, b)$ is the distance between points a and b. Intuitively, the point p represents a location of the post office that minimizes average distance.

(d) Argue that the weighted median is a best solution for the 1-dimensional post office location problem, in which points are simply real numbers and the distance between points a and b is $d(a, b) = |a - b|$.

(e) Find the best solution for the 2-dimensional post office location problem, in which the points are (x, y) coordinate pairs and the distance between points $a = (x_1, y_1)$ and $b = (x_2, y_2)$ is the Manhattan distance given by $d(a, b) = |x_1 - x_2| + |y_1 - y_2|$.

5. Show that the smallest and the second smallest of n distinct elements can be found with $n + \lceil \lg n \rceil - 2$ comparisons in the worst case.