Today’s topics

Computer Hardware
   Electric Circuits
   Designing an Adder

Upcoming
   Computer Communications
      (Great Ideas Chapter 10)

Reading
   (not in text)
Binary Addition ($Z = X + Y$)

- Like Decimal, but---
  - Have only two symbols: 0, 1
- At first, seems like *two* “inputs” will do
  
  $X: \quad 10010$
  
  $Y: \quad +01001$
  
  $Z: \quad 11011$

- Looking at it
  - From right: $0+1 = 1; 1+0 = 1; 0+0 = 0; 0+1 = 1; 1+0 = 1$
  - However, example not realistic
  - Must deal with possible carries
  - Need better example
Binary Addition \((Z = X + Y)\) (+carry)

- Let’s try
  
  \[
  \begin{align*}
  C & : 001100100 \\
  X & : 100110011 \\
  Y & : +000110010 \\
  Z & : 101100101
  \end{align*}
  \]

- Must add a top row for carries to get whole picture

- To add two number (by columns) takes *three* inputs
  
  - \(X, Y\) and \(C\) (for carry)
  - So, *from right*: \(0+0+1 = 1\) (carry 0); \(0+1+1 = 0\) (carry 1); \(1+0+0 = 1\) (carry 0); \(0+1+1 = 0\) (carry 1); \(0+0+0 = 0\) (carry 0); \(0+1+1 = 0\) (carry 1); \(1+1+1 = 1\) (carry 1); \(1+0+0 = 1\) (carry 0); \(0+0+0 = 0\) (carry 0); \(0+1+0 = 1\) (carry 0)
Truth Tables for Addition

- We need two 3-input truth tables
  - One for the resulting *Sum* bit
  - One for the resulting *Carry* bit

**Sum:**

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>C</th>
<th>S</th>
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<tbody>
<tr>
<td>0</td>
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$$S = X'Y'C + X'YC' + XY'C' + XYC$$
Truth Tables for Addition

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>C</th>
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</table>

\[ C = X'Y'C + X'Y'C + XY'C' + X'Y'C \]
The 4 bit Adder

- Now have the building-blocks to put together an Adder of arbitrary size
- Design in several steps (illustrated by drawings on web page)
  1. Block Diagram
  2. Simple Adder
  3. Control Section
  4. Putting it all together: The 4 Bit Adder
- Will be on quizzes and/or Final Exam
- Learn how to go through circuits and *mark them*
- May encounter different circuits
  - E.g., a Subtractor
  - Same marking methods will apply