Today’s Topics

Computer Science
   Program Execution Time:
      Intractable Algorithms

Upcoming
   Parallel Computing
      Great Ideas, Chapter 14

Reading
   Great Ideas, Chapter 13
On the Limits of Computing

• Intractable Algorithms
  - Computer "crawls" or seems to come to halt for large N
  - Large problems *essentially unsolved*
  - May never be able to compute answer for some obvious questions

• Chess
  - Note: here N is number of moves looking ahead
  - We Have an Algorithm!
    - Layers of look-ahead: If I do this, then he does this, ....
    - Problem Solved (?!)
  - Can Represent Possibilities by Tree
  - Assume 10 Possibilities Each Move
  - \( t = A \times 10^N \)

• Exponential !!!
Exponential Algorithms

- Sample Numbers
  - Get BIG very rapidly
  - Numbers seem to EXPLODE
  - At each step, amount of work multiplies rather than adds
- Exponential = Intractable
- Traveling Salesperson Example
  - Visit N Cities in Optimal Order
  - Optimize for minimum:
    - Time
    - Distance
    - Cost
- N factorial (N!) Possibilities
- N! is (very) roughly N^N
  - Stirling’s approximation: N! = sqrt(2*Pi*N)\*(N/e)^N
- Typical of some very practical problems
Traveling Salesperson Examples

- 3 cities $2! = 2$ possible routes (1 of interest)
  - abc
  - acb
- 4 cities $3! = 6$ possible routes (3 of interest)
  - abcd
  - abdc
  - acbd
  - acdb
  - acbd
  - adbc
  - adcb

- Only half usually of interest because just reverse of another path
# Traveling Salesperson Examples

5 cities 4! = 24 possible routes

- abcde
- abced
- abdce
- abdec
- abecd
- abedc
- acbde
- acbed
- acdbe
- acdeb
- acebd
- acedeb
- acedbc

(12 of interest)

- adbce
- adbce
- adcbe
- adceb
- adebc
- adecb
- aedbc
- aedcb
## Towers of Hanoi

The Towers of Hanoi is a mathematical puzzle where $N$ discs of varying radii are moved from one of three pegs to another, with the rule that a larger disc cannot be placed on a smaller one.

The time $t$ it takes to solve the puzzle, for a very old PC, is given by the formula:

$$t = 0.00549 \times 2^N$$

<table>
<thead>
<tr>
<th>$N$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>.17 sec</td>
</tr>
<tr>
<td>10</td>
<td>5.62 sec</td>
</tr>
<tr>
<td>15</td>
<td>3.00 min</td>
</tr>
<tr>
<td>20</td>
<td>1.6 hour</td>
</tr>
<tr>
<td>25</td>
<td>2.13 day</td>
</tr>
<tr>
<td>30</td>
<td>68.23 day</td>
</tr>
<tr>
<td>35</td>
<td>5.98 year</td>
</tr>
<tr>
<td>40</td>
<td>191.3 year</td>
</tr>
<tr>
<td>45</td>
<td>6120 year</td>
</tr>
<tr>
<td>50</td>
<td>196 K year</td>
</tr>
<tr>
<td>55</td>
<td>6.27 M year</td>
</tr>
<tr>
<td>60</td>
<td>201 M year</td>
</tr>
<tr>
<td>65</td>
<td>6.42 G year</td>
</tr>
<tr>
<td>70</td>
<td>205 G year</td>
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Intractable Algorithms

- Other Games
- More hardware not the answer!
- Predicting Yesterday's Weather
- Actual Examples for Time Complexity