Today's Topics

Computer Science
Program Execution Time (Performance!)

Upcoming
Parallel Computing
Great Ideas, Chapter 14

Reading
Great Ideas, Chapter 13

On the Limits of Computing

- Reasons for Failure
  1. Runs too long
     - Real time requirements
     - Predicting yesterday's weather
  2. Non-computable
  3. Don't know the algorithm

- Complexity, N
  - Time
  - Space

- Tractable and Intractable

- Study of a Sorting Algorithm
  - Sorting, Ordering
  - Alphabetizing

Sorting Example

- Selection Sort
  - N items in an array named Data
    - [ 2 | 4 | 7 | 3 | 1 | 8 | 5 ]
  - Find smallest of elements 1 thru N of Data
  - Interchange this with 1st element of array Data
    - [ _ | _ | _ | _ | _ | _ | _ ]
  - Find smallest of elements 2 thru N of Data
  - Interchange this with 2nd element of array Data
    - [ _ | _ | _ | _ | _ | _ | _ ]
  - ...
  - Find smallest of elements K thru N of Data
  - Interchange this with Kth element of array Data
    - [ _ | _ | _ | _ | _ | _ | _ ]
    - [ _ | _ | _ | _ | _ | _ | _ ]
    - [ _ | _ | _ | _ | _ | _ | _ ]
  - Done when K = N
    - [ _ | _ | _ | _ | _ | _ | _ ]

Sorting Example

- Selection Sort
  - N items in an array named Data
    - [ 2 | 4 | 7 | 3 | 1 | 8 | 5 ]
  - Find smallest of elements 1 thru N of Data
  - Interchange this with 1st element of array Data
    - [ 1 | 4 | 7 | 3 | 2 | 8 | 5 ]
  - Find smallest of elements 2 thru N of Data
  - Interchange this with 2nd element of array Data
    - [ 1 | 2 | 7 | 3 | 4 | 8 | 5 ]
  - ...
  - Find smallest of elements K thru N of Data
  - Interchange this with Kth element of array Data
    - [ 1 | 2 | 3 | 7 | 4 | 8 | 5 ]
    - [ 1 | 2 | 3 | 4 | 7 | 8 | 5 ]
    - [ 1 | 2 | 3 | 4 | 5 | 8 | 7 ]
  - Done when K = N
    - [ 1 | 2 | 3 | 4 | 5 | 7 | 8 ]
Analysis of Sorting Example

❖ How Many Operations?
  ❖ Comparisons
  ❖ N-1 comparisons in first pass
  ❖ N-2 comparisons in first pass
  ❖ ...
  ❖ 1 comparisons in last pass
  ❖ N-1 + N-2 + N-3 + ... 2 + 1
  ❖ N*(N-1)/2 = N*N/2 - N/2 (Gauss)
❖ What does Order N Square Mean?
  ❖ Examples

<table>
<thead>
<tr>
<th>N</th>
<th>N*N/2</th>
<th>-N/2</th>
<th>comparisons</th>
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<td>2</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
<td>-1.5</td>
<td>3</td>
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<tr>
<td>4</td>
<td>8</td>
<td>-2</td>
<td>6</td>
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<tr>
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<td>18</td>
<td>-3</td>
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<td>-4</td>
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<td>-50</td>
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<tr>
<td>1000</td>
<td>500000</td>
<td>-500</td>
<td>499500</td>
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❖ t = A * N * N = A * N^2

Polynomial Time

❖ Linear Time Algorithms
  ❖ Add elements of an array
  ❖ Single loop algorithms
  ❖ t = A * N
❖ Cubic Time Algorithms
  ❖ Matrix multiplication
  ❖ t = A * N^3
❖ Polynomial Time
  ❖ t = A * N^K
  ❖ ... and in-between
  ❖ Faster machines make a lot of difference
❖ Quicksort
  ❖ t = A * N * log(N)
  ❖ Logarithmic behavior

Polynomial Time

❖ What does Order log(N) or N*log(N) Mean?
  ❖ Various values of N

<table>
<thead>
<tr>
<th>N</th>
<th>log(N)</th>
<th>N*log(N)</th>
<th>N^2</th>
</tr>
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<tbody>
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<td>2</td>
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<td>1M</td>
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<td>20</td>
<td>20M</td>
<td>1T</td>
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<tr>
<td>2M</td>
<td>21</td>
<td>42M</td>
<td>4T</td>
</tr>
</tbody>
</table>

❖ K = 1024;  M = K * K;  G = K * M;  T = K * G
Tractable Algorithms

- Graphs Showing Complexity
- Polynomial = Tractable

- Binary Search
  - Assumes Sorted
  - Like telephone book lookup
  - Logarithmic Time
  - \( t = A \times \log(N) \)

- Intractable Algorithms
  - Computer "crawls" or seems to come to halt for large \( N \)
  - Large problems essentially unsolved
  - May never be able to compute answer for some obvious questions