On the Limits of Computing

- **Intractable Algorithms**
  - Computer "crawls" or seems to come to halt for large N
  - Large problems *essentially unsolved*
  - May never be able to compute answer for some obvious questions

- **Chess**
  - Note: here N is number of moves looking ahead
  - We *have* an Algorithm!
    - Layers of look-ahead: If I do this, then he does this, ...!
    - Problem Solved (?!)
  - Can Represent Possibilities by Tree
  - Assume 10 Possibilities Each Move
  - \( t = A \times 10^N \)

- **Exponential !!!**

Exponential Algorithms

- **Recognizing Exponential Growth**
  - Things get **BIG** very rapidly
  - Numbers seem to **EXPLODE**
  - **KEY**: at each added step, work **multiplies** rather than **adds**

- **Exponential = Intractable**

- **Traveling Salesperson Example**
  - Visit N Cities in *Optimal Order*
  - Optimize for minimum:
    - Time
    - Distance
    - Cost

- **N factorial (N!) Possibilities**
  - \( N! \) is (very) roughly \( N^N \)
  - Stirling’s approximation: \( N! = \sqrt{2\pi N} \times (N/e)^N \)

- **Typical of some very practical problems**

Traveling Salesperson Examples

- **3 cities**: \( 2! = 2 \) possible routes (1 of interest)
  - abc
  - acb

- **4 cities**: \( 3! = 6 \) possible routes (3 of interest)
  - abcd
  - abdc
  - acbd
  - acdb
  - adbc
  - adb

- (Only half usually of interest because just reverse of another path)
Traveling Salesperson Examples

5 cities 4! = 24 possible routes (12 of interest)

- abede
- abced
- abdce
- abdec
- abced
- abdec
- acbde
- acbed
- acdbe
- acebd
- acebd
- acedc
- acedc
- adecb
- adecb
- adecb
- adecb

Towers of Hanoi

\[ t = 0.00549 \times 2^N \]

(for a very old PC)

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<th>( N )</th>
<th>( t )</th>
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<td>.17 sec</td>
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<tr>
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<tr>
<td>15</td>
<td>3.00 min</td>
</tr>
<tr>
<td>20</td>
<td>1.6 hour</td>
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<tr>
<td>25</td>
<td>2.13 day</td>
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<td>6.42 G year</td>
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<td>70</td>
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Intractable Algorithms

- Other Games
- More hardware not the answer!
- Predicting Yesterday's Weather
- Actual Examples for Time Complexity