Today’s topics

**Computer Hardware**
  Electric Circuits
  Designing an Adder

**Upcoming**
  Computer Communications
  (*Great Ideas* Chapter 10)

**Reading**
  *(not in text)*
Binary Addition (Z = X + Y)

- Like Decimal, but---
  - Have only two symbols: 0, 1

- At first, seems like *two* "inputs" will do
  x: 10010
  y: +01001
  z: 11011

- Looking at it
  - From right: 0+1 = 1; 1+0 = 1; 0+0 = 0; 0+1 = 1; 1+0 = 1
  - However, example *not realistic*
  - Must deal with possible *carries*
  - Need better example
Binary Addition \((Z = X + Y)\) (+carry)

- **Let’s try**
  
  \[
  \begin{align*}
  C & : \quad 001100100 \\
  X & : \quad 100110011 \\
  Y & : \quad +000110010 \\
  \hline
  Z & : \quad 101100101
  \end{align*}
  \]

- **Must add a top row for carries to get whole picture**

- **To add two number (by columns) takes three inputs**
  
  - X, Y and C (for carry)
  - **So, from right:**
    
    \[
    \begin{align*}
    0+0+1 &= 1(\text{carry 0}); \\ 0+1+1 &= 0(\text{carry1}); \\ 1+0+0 &= 1(\text{carry 0}); \\ 0+1+1 &= 0(\text{carry 1}); \\ 0+0+0 &= 0(\text{carry0}); \\ 0+1+1 &= 0(\text{carry 1}); \\ 1+1+1 &= 1(\text{carry 1}); \\ 1+0+0 &= 1(\text{carry0}); \\ 0+0+0 &= 0 (\text{carry 0}); \\ 0+1+0 &= 1 (\text{carry 0})
    \end{align*}
    \]
Truth Tables for Addition

- We need two 3-input truth tables
  - One for the resulting *Sum* bit
  - One for the resulting *Carry* bit

**Sum:**

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>C</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</table>

\[ S = X'Y'C + X'YC' + XY'C' + XYC \]
# Truth Tables for Addition

## Carry:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>C</th>
<th>C</th>
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</thead>
<tbody>
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<td>0</td>
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</tbody>
</table>

\[
C = X'YC + XY'C + XYC' + XYC
\]

Diagram: [Circuit Diagram for Carry](#)
The 3 bit Adder

- Now have the building-blocks to put together an Adder of arbitrary size
- Design in several steps (illustrated by drawings on web page)
  1. Block Diagram
  2. Simple Adder
  3. Control Section
  4. Putting it all together: The 3 Bit Adder
- Will be on quizzes and/or Final Exam
- Learn how to go through circuits and mark them
- May encounter different circuits
  - E.g., a Subtractor
  - Same marking methods will apply