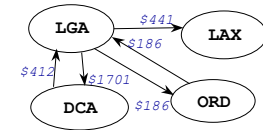
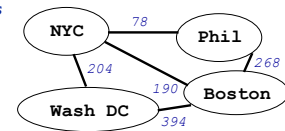


Graphs: Structures and Algorithms

- How do packets of bits/information get routed on the internet
 - Message divided into packets on client (your) machine
 - Packets sent out using routing tables toward destination
 - Packets may take different routes to destination
 - What happens if packets lost or arrive out-of-order?
 - Routing tables store local information, not global (why?)
- What about [The Oracle of Bacon](#), [Erdos Numbers](#), and [Word Ladders](#)?
 - All can be modeled using graphs
 - What kind of connectivity does each concept model?
- Graphs are everywhere in the world of algorithms (world?)

Vocabulary

- Graphs are collections of *vertices* and *edges* (vertex also called node)
 - Edge connects two *vertices*
 - Direction can be important, *directed edge*, *directed graph*
 - Edge may have associated weight/cost
- A vertex sequence v_0, v_1, \dots, v_{n-1} is a *path* where v_k and v_{k+1} are connected by an edge.
 - If some vertex is repeated, the path is a *cycle*
 - A graph is *connected* if there is a path between any pair of vertices



Graph questions/algorithms

- What vertices are reachable from a given vertex?
 - Two standard traversals: depth-first, breadth-first
 - Find *connected components*, groups of connected vertices
- Shortest path between any two vertices (weighted graphs?)
 - Breadth first search is storage expensive
 - Dijkstra's algorithm is efficient, uses a priority queue too!
- Longest path in a graph
 - No known efficient algorithm
- Visit all vertices without repeating? Visit all edges?
 - With minimal cost? Hard!

Depth, Breadth, other traversals

- We want to visit every vertex that can be reached from a specific starting vertex (we might try all starting vertices)
 - Make sure we don't visit a vertex more than once
 - Why isn't this an issue in trees?
 - Mark vertex as visited, use set/array/map for this
 - Can keep useful information to help with visited status
 - Order in which vertices visited can be important
 - Storage and runtime efficiency of traversals important
- What other data structures do we have: stack, queue, ...
 - What happens when we traverse using priority queue?

Breadth first search

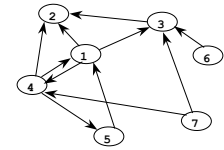
- In an unweighted graph this finds the shortest path between a start vertex and every vertex
 - Visit every node one away from start
 - Visit every node two away from start
 - This is every node one away from a node one away
 - Visit every node three away from start, ...
- Put vertex on queue to start (initially just one)
 - Repeat: take vertex off queue, put all adjacent vertices on
 - Don't put a vertex on that's already been visited (why?)
 - When are 1-away vertices enqueued? 2-away? 3-away?
 - How many vertices on queue?

CPS 100

16.9

Code for breadth first

```
public void breadth(String vertex){
    Set visited = new TreeSet();
    LinkedList q = new LinkedList();
    q.addLast(vertex);
    visited.add(vertex);
    while (q.size() > 0) {
        String current = (String) q.removeFirst();
        // process current
        for(each v adjacent to current){
            if (!visited.contains(v)) { // not visited
                visited.add(v);
                q.addLast(v);
            }
        }
    }
}
```

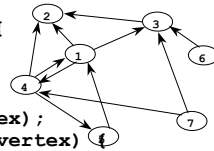


CPS 100

16.10

Pseudo-code for depth-first search

```
void depthfirst(String vertex){
    if (! alreadySeen(vertex))
    {
        markAsSeen(vertex);
        System.out.println(vertex);
        for(each v adjacent to vertex)
            depthfirst(v);
    }
}
```



- Clones are stacked up, problem? Can we make use of stack explicit?

CPS 100

16.11

Edsger Dijkstra

- Turing Award, 1972
 - Operating systems and concurrency
 - Algol-60 programming language
 - Goto considered harmful
 - Shortest path algorithm
 - Structured programming
- "Program testing can show the presence of bugs, but never their absence"*
- A Discipline of programming
- "For the absence of a bibliography I offer neither explanation nor apology"*



CPS 100

16.12

Schedule students, minimize conflicts

- Given student requests, available teachers
 - write a program that schedules classes
 - Minimize conflicts
- Add a GUI too
 - Web interface
 - ...
 - ...



CPS 100

16.21

One better scenario



CPS 100

16.22

Another possible scenario



CPS 100

16.23

Entscheidungsproblem

- What can we program?
- What can't we program?
- Can we write a program that will determine if any program P will halt when run on input S ?
 - Input to halt: P and S
 - Output: yes/no halts



CPS 100

16.24

The halting problem: writing `doesHalt`

```
public class ProgramUtils
/**
 * Returns true if progname halts on input,
 * otherwise returns false (progname loops)
 */
public static boolean doesHalt(String progname,
                               String input){
}
}
```

- A compiler is a program that reads other programs as input
 - Can a word counting program count its own words?
- The `doesHalt` method might simulate, analyze, ...
 - One program/function that works for *any* program/input

How to tell if Foo stops on 123 456

```
public static void main(String[] args) {
    String prog = "Foo.java";
    String input = "123 456"
    if (ProgramUtils.doesHalt(prog,input)) {
        System.out.println(prog+" stops");
    }
    else {
        System.out.println(prog+" 4ever");
    }
}
```

- Can user enter name of program? Input?
 - What's the problem with this program?

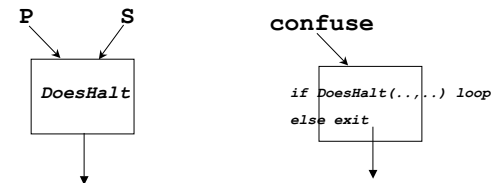
Consider the class `Confuse.java`

```
public static void main(String[] args){
    String prog = "Foo.java";
    if (ProgramUtils.doesHalt(prog,prog)) {
        while (true) {
            // do nothing forever
        }
    }
}
```

- We want to show writing `doesHalt` is impossible
 - Proof by contradiction:
 - Assume possible, show impossible situation results
- Can a program read a program? Itself?

Can we write `Confuse.java`?

- Legal if `doesHalt` exists
 - What have we assumed?
- What are consequences of running `confuse` on itself?
 - Trouble?



What's a meta catalog? Top 10 sites?

- Consider a website of interesting sites
 - Does the website list itself? Is this a problem?
- Consider a website that lists every useless website
 - Would this be a useful resource?
 - Does the website list itself?
- What about a site of all the sites that list themselves?
 - What about sites that don't list themselves? *noList.com*

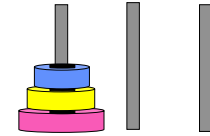


CPS 100

16.29

Not impossible, but impractical

- Towers of Hanoi
 - How long to move n disks?
- What combination of switches turns the light on?
 - Try all combinations, how many are there?
 - Is there a better way?

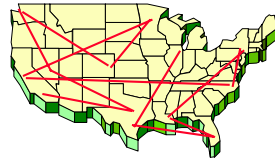


CPS 100

16.30

Travelling Salesperson

- Visit every city exactly once
- Minimize cost of travel or distance
- Is there a tour for under \$2,000 ? less than 6,000 miles?
- Is close good enough?
 - Within 10% of optimal
 - Within 50% of optimal
 - ...



Try all paths, from every starting point -- how long does this take?

a, b, c, d, e, f, g
b, a, c, d, e, f, g ...

CPS 100

16.31

Are hard problems easy?

- P = easy problems, NP = "hard" problems
 - P means solvable in polynomial time
 - Difference between N , N^2 , N^{10} ?
 - NP means non-deterministic, polynomial time
 - *guess a solution and verify it efficiently*
- Question: P = NP ?
 - if yes, a whole class of difficult problems, the NP-complete problems, can be solved efficiently
 - if no, none of the hard problems can be solved efficiently
 - showing the first problem was NP complete was an exercise in intellectual bootstrapping, satisfiability/Cook/(1971)

CPS 100

16.32

Theory and Practice

- **Number theory: pure mathematics**
 - How many prime numbers are there?
 - How do we factor?
 - How do we determine primeness?
- **Computer Science**
 - Primality is “easy”
 - Factoring is “hard”
 - Encryption is possible



public-key cryptography
randomized primality
testing

Computer Science in a Nutshell

The Google logo, consisting of the word "Google" in its characteristic multi-colored font.

Web Images Groups News Froogle more »
Google Search | I'm Feeling Lucky



[Advertising Programs](#) · [Business Solutions](#) · [About Google](#)

©2004 Google - Searching 8,058,044,851 web pages