COMPSCI 270 - Artificial Intelligence
Homework Assignment II

Due date: Sep 24

1 Basic Probability

Define a probability distribution over three random variables, A, B, and C, such that A and B are independent, B and C are independent, but A and C are *not* independent. (Hint: It's easier if you start thinking about A and C first.)

2 Conditional Probability

Prove the extended form of Bayes's rule:

$$P(A|BC) = \frac{P(B|AC)P(A|C)}{P(B|C)}$$

3 Expectation

Suppose you have two dice labeled 1 and 2. (i) If these are fair dice, what is the expected value of the sum of the pips when you roll the two dice? (ii) Suppose die 2 is rigged so that it produces the exact same outcome as die 1. How does this change the expected value of their sum? (iii) Suppose die 2 is rigged so that it produces the value of die 1 plus one (wrapping back to one when die 1 produces a 6). How does this change the expected value of their sum?

4 Decision Theory

Suppose you're a contestant in a trivia game show that involves answering 3 questions with values of 1000, 50000, and 250000. If you answer a question correctly, you proceed to the next higher valued question. At each stage, you have the option of taking your winnings thus far and quitting, or proceeding to the next question. Each question is multiple choice with 5 possible answers. You aren't very good at trivia questions, so your chance of getting any question right is no better than guessing. If you miss a question, you forfeit all winnings and the game ends, so you need to think carefully about whether it's worth proceeding. (i) What is the optimal strategy for playing this game and what is its expected payoff? (ii) Suppose that you have the option of asking for help at some point in the game. The help will come in the form of a hint that eliminates one of the five possible answers. If you can only ask for help once, what is the optimal point at which to ask for help, and how much does this increase your expected payoff for playing the game?

5 Value of Information I

Give a plausible example of a real life situation with real numbers where the value of information is *not* additive. Work through the numbers to demonstrate clearly that it is not additive.

6 Costs with loops

Suppose a robot with a defective motor is moving down a narrow corridor. At each time step, the robot attempts to move forward by 1 meter. With probability 0.7, the robot will successfully move forward by 1 meter. With probability 0.2, the robot will stay where it is, and with probability 0.1 it will move backwards by 1 meter. (If the robot's motor sends it backward from the initial state, it will bump into a wall and stay in it's initial position, so you we can think of the start state as a special case where the robot has probability 0.3 of staying put.)

If the robot's goal is 5 meters away and it consumes 1 unit of battery power for each attempted movement, what is the expected cost in battery power for the robot to reach its goal?