CPS130: Homework 4
Due: Tuesday, November 8

1. A connected graph is vertex biconnected if there is no vertex whose removal disconnects the graph. A connected graph is edge biconnected if there is no edge whose removal disconnects the graph. Give a proof or counterexample for each of the following statements (consider only the undirected graphs having more than 1 vertex):

   a) A vertex biconnected graph is edge biconnected.

   b) An edge biconnected graph is vertex biconnected.

2. Consider a connected undirected graph with edge weights. We call a spanning tree a *minimax spanning tree*, or *MMST* if its longest edge is at most as long as the longest edge of any other spanning tree.

   a) Is every MST an MMST? Why or why not?

   b) How about the other way around: Is every MMST an MST? Explain why or why not.

3. Suppose we are given a directed graph $G = (V, E)$ on which each edge (u, v) is given a real value $r(u, v)$ in the range $[0, 1]$, which represents the reliability of a communication channel. We can interpret this value as the probability that the edge from u to v will fail, and we moreover assume these probabilities are independent. Give an efficient algorithm to find the most reliable path between any two vertices, s and t. *Hint: Consider how to compute the total reliability of a path. Is it a sum or a product?*