CPS130 - Homework 5
Due: Tuesday, Nov 29

1. Consider this game. We are playing on an n x n board with a number in each square, possibly negative. When you visit a square, you get points equal to the number on that square. Each square also has three moves associated with it, that is, three squares to which you can visit from here. These moves can go to any square on the board, not only to ones immediately adjacent to the square; for example, the square $(1,3)$ could go to $(4,9)$, $(1,2)$, or $(0,1)$. Our goal is to get from the square $(0,0)$ to the square (n,n) with the largest possible score. Assume that you cannot ever return to a square once you leave.

   a) Give a greedy (or myopic) approach to solve this problem. Is this optimal? Give an example where it isn't and explain what went wrong.

   b) Give a simple recursive procedure to find the maximal path, without any use of memory. What is the time complexity of this approach?

   c) Give an algorithm that uses dynamic programming. How much faster is this approach?

   d) Since you can't return to a square once you leave, we can model this problem as a graph. There is one node for each box and the three transitions represent edges leaving the node. Give a graph theoretic algorithm to find the optimal set of moves. Is this any faster than the dynamic programming approach?

2. Suppose you're planning on going on a backpacking trip in the mountains over Thanksgiving break. Having been training all fall for the trip, you think you can carry at most C pounds worth of food. Your goal is to pick the best food to take with you on the trip, where each food item has some caloric value c_i and you only have (at most) a_i pounds of it. Give a greedy strategy if:

   a) You can choose to take some fraction of each food item, that is, you're deciding the percentage of each item to take to maximize the total caloric value. (Note that you cannot, however, get any more food - these values have to be between 0 and 1)

   b) All of the food is in sealed containers, so you must take all a_i pounds if you take any of it. You still want the maximal caloric value, however, but your percentages must be either 0 or 100.

   Discuss briefly whether or not each strategy is optimal. Give a proof if it is optimal, or a counter example if it isn't.

3. Describe a generalization for the FFT where n is a power of 3 instead of 2. Analyze the running time of the new algorithm.