Part I.

1. Design a method for computing division with addition and multiplication only, within a specified error tolerance.
   Estimate the computation cost per iteration and the number of iterations in terms of the tolerance.

2. Design a division-free method for extracting the primary square root within a given error tolerance and with quadratic convergence rate.
   Provide a convergence analysis.
   Estimate the computation cost per iteration and the number of iterations in terms of the tolerance.

3. Design a method for extracting the primary root \( r^{1/4}, r > 0 \). Provide a convergence analysis.
   Estimate the computation cost per iteration and the number of iterations in terms of the tolerance.
   Extend the method for the root extraction of \( \alpha^{1/4} \) with arbitrary nonzero \( \alpha \).

4. Consider numerical solution to the following system of nonlinear equations
   \[
   \begin{align*}
   \delta_1 v + \alpha_1 v^3 + \beta_1 w * v + \kappa_1 &= 0 \\
   \gamma_2 v + \beta_2 w + \kappa_2 &= 0
   \end{align*}
   \]
   where the letters in Greek denote constant parameters.
   Discuss the issues in Greek denote constant parameters.
   Discuss the issues in the design and development of a numerical solution method.

5. Consider numerical integration of the following I.V. problem
   \[
   \frac{dv}{dt} = v(v - \text{threshold})(\text{peak} - v), \quad t \in (t_0, T)
   \]
   with the initial value \( v(t_0) \) provided. Design a scheme that allows automatic adaptation in integration step size.
   [Optional.] Incorporate adaptive selection between the forward and backward Euler methods.

6. Discretize the following derivatives of a function \( f(t, x, y) \) at equally spaced sampling points.
   \[
   \begin{align*}
   \frac{\partial f}{\partial t}, & \quad \frac{\partial f}{\partial x}, & \quad \frac{\partial f}{\partial y}, & \quad \frac{\partial^2 f}{\partial x \partial x}, & \quad \frac{\partial^2 f}{\partial y \partial y}, & \quad \frac{\partial^2 f}{\partial x \partial y}.
   \end{align*}
   \]
Part II.

1. Provide matlab implementation of your root extraction algorithms in Prob. 2 and 3, Part I.

2. Provide in matlab the image of Newton’s basin sets for the equation $z^4 + 4 = 0$.

3. Provide matlab implementation of your adaptive integration method for Prob. 5 in Part I.

   Make a comparison with the algorithm using the forward Euler and uniform step size (as in vcell\_demo.m).

4. Provide an improvement to the code in cell\_torus2.m. Make a comparison.