PROBLEM 1 : (***Family Trees***)

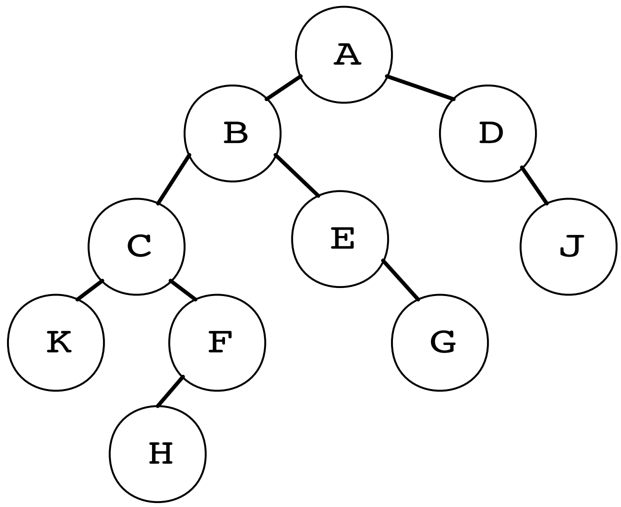
In this problem assume all values in trees are unique, no value appears more than once in a tree. **In this problem the tree is not necessarily a search tree**.

The code in the function `leastAncestor` shown below returns a pointer to the least ancestor of two strings in a tree.

The *least ancestor* of two string values `p` and `q` is the node furthest from the root (deepest) which is an ancestor of both `p` and `q` (there is a path from the least ancestor to both `p` and `q`).

For example, the tree diagrammed below on the right yields the values shown in the table on the left.

`p`	`q`	**least ancestor**
F	E	B
C	G	B
K	H	C
H	J	A
A	G	A
G	A	A



Part A (8 points)

The complexity of *leastAncestor* shown on the next page is not $O(n)$ for an n-node tree. What is the complexity and why? Justify using recurrence relations and an explanation of each part of the recurrence. Provide big-Oh complexities in the average case (assume trees are roughly balanced) and the worst case.

Here's code to find the least ancestor, the helper method `inTree` is called from method `leastAncestor`. Note (again) that in this problem trees are not search trees.

```
public static boolean inTree(TreeNode root, String s){
    if (root == null) return false;
    if (root.info.equals(s)) return true;
    return inTree(root.left,s) || inTree(root.right,s);
}

public static TreeNode leastAncestor(TreeNode t, String p, String q){

    if (t == null) return null;

    // first check subtrees (lower than me) for ancestor

    TreeNode result = leastAncestor(t.left, p,q);
    if (result != null) return result;

    result = leastAncestor(t.right,p,q);
    if (result != null) return result;

    // didn't find in subtrees, am I the least ancestor? check
    // me and left/right subtrees for p/q (vice versa)

    if ( (t.info.equals(p) || inTree(t.left,p))  &&
         (t.info.equals(q) || inTree(t.right,q))) {
        return t;
    }
    if ( (t.info.equals(q) || inTree(t.left,q))  &&
         (t.info.equals(p) || inTree(t.right,p))) {
        return t;
    }

    return null;
}
```

Part B (6 points)

Write the function *findPath* whose header is given below. The function sets values in/returns an `ArrayList` representing the path from the root of `t` to the node containing `target` if there is a path, or returns an empty vector otherwise.

For example, given the tree on the previous page we have:

call	**ArrayList list**
`findPath(t,"F", list)`	`(A,B,C,F)`
`findPath(t,"A", list)`	`(A)`
`findPath(t,"J", list)`	`(A,D,J)`
`findPath(t,"G", list)`	`(A,B,E,G)`
`findPath(t,"B", list)`	`(A,B)`
`findPath(t,"X", list)`	`()`

```
/**
  * Add values in list so that they represent strings
  * on path from t to node containing target. If target
  * not in the tree then no values added to list
  */

public static void findPath(TreeNode t, String target, ArrayList<String> list) {
    if (t == null) return;
    if (t.info.equals(target)){
        list.add(t.info);
        return;
    }
    // add code here

}
```

Part C (8 points)

Write a version of `leastAncestor` that runs in $O(n)$ time for an n-node tree. You can use any approach; one approach is to use the function `findPath` from part B, another is to write an auxiliary function that returns three values (e.g., in an array or list) an ancestor-pointer and a boolean that tells if p is in the tree and a boolean that tells if q is in the tree.

Write the code and justify that it runs in $O(n)$ time.