Analyzing Algorithms

- Consider three solutions to SortByFreqs
  - Sort, then scan looking for changes
  - Insert into Set, then count each unique string
  - Find unique elements without sorting, sort these, then count each unique string
  - Use a Map (TreeMap or HashMap)
- We want to discuss trade-offs of these solutions
  - Ease to develop, debug, verify
  - Runtime efficiency
  - Vocabulary for discussion

What is big-Oh about? (preview)

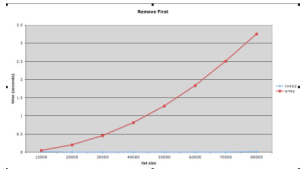
- Intuition: avoid details when they don't matter, and they don't matter when input size (N) is big enough
  - For polynomials, use only leading term, ignore coefficients

`y = 3x` `y = 6x-2` `y = 15x + 44`
`y = x^2` `y = x^2-6x+9` `y = 3x^2+4x`

- The first family is `O(n)`, the second is `O(n^2)`
  - Intuition: family of curves, generally the same shape
  - More formally: `O(f(n))` is an *upper-bound*, when n is large enough the expression `cf(n)` is larger
  - Intuition: linear function: double input, double time, quadratic function: double input, quadruple the time

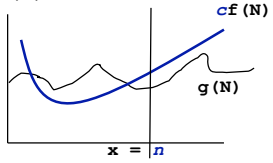
Recall adding to list (class handout)

- Add one element to front of ArrayList
  - Shift all elements
  - Cost *N* for *N*-element list
  - Cost 1 + 2 + … + N = N(N+1)/2 if repeated
- Add one element to front of LinkedList
  - No shifting, add one link
  - Cost is independent of *N*, *constant-time* cost
  - Cost 1 + 1 + … + 1 = N if repeated

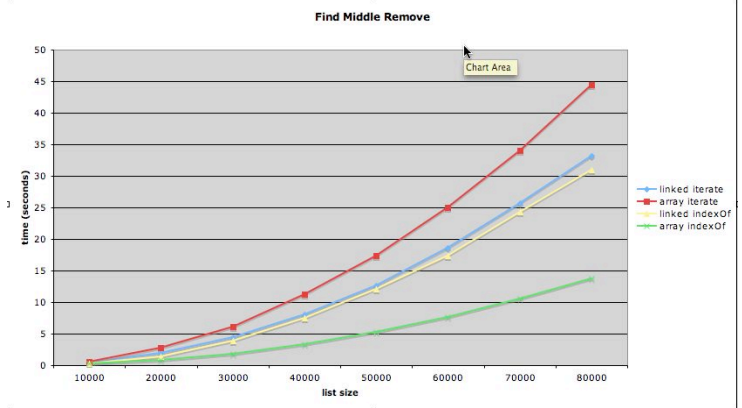


More on O-notation, big-Oh

- Big-Oh hides/obscures some empirical analysis, but is good for general description of algorithm
  - Allows us to compare algorithms *in the limit*
    - 20N hours vs N^2 microseconds: *which is better?*
- O-notation is an upper-bound, this means that `N` is `O(N)`, but it is also `O(N^2)`; we try to provide *tight* bounds. Formally:
  - A function `g(N)` is `O(f(N))` if there exist constants *c* and *n* such that `g(N) < cf(N)` for all `N > n`



Which graph is "best" performance?



Big-Oh calculations from code

- Search for element in an array:
  - What is complexity of code (using O-notation)?
  - What if array doubles, what happens to time?

```
for(int k=0; k < a.length; k++) {
   if (a[k].equals(target)) return true;
};
return false;
```

- Complexity if we call N times on M-element vector?
  - What about best case? Average case? Worst case?

Amortization: Expanding ArrayLists

- Expand capacity of list when `add()` called
- Calling `add` N times, doubling capacity as needed

Item #	Resizing cost	Cumulative cost	Resizing Cost per item	Capacity After add
1	0	0	0	1
2	2	2	1	2
3-4	4	6	1.5	4
5-8	8	14	1.75	8
2^m+1 - 2^{m+1}	2^{m+1}	$2^{m+2}-2$	around 2	2^{m+1}

- What if we grow size by one each time?

Some helpful mathematics

- $1 + 2 + 3 + 4 + \ldots + N$
  - $N(N+1)/2$, exactly = $N^2/2 + N/2$ which is $O(N^2)$ why?
- $N + N + N + \ldots + N$ (total of N times)
  - $N*N = N^2$ which is $O(N^2)$
- $N + N + N + \ldots + N + \ldots + N + \ldots + N$ (total of 3N times)
  - $3N*N = 3N^2$ which is $O(N^2)$
- $1 + 2 + 4 + \ldots + 2^N$
  - $2^{N+1} - 1 = 2 \times 2^N - 1$ which is $O(2^N)$
- Impact of last statement on adding 2^N+1 elements to a vector
  - $1 + 2 + \ldots + 2^N + 2^{N+1} = 2^{N+2}-1 = 4x2^N-1$ which is $O(2^N)$
    `resizing + copy = total (let x = 2^N)`

Running times @ 10^6 instructions/sec

N	*O(log N)*	*O(N)*	*O(N log N)*	*O(N^2)*
10	0.000003	0.00001	0.000033	0.0001
100	0.000007	0.00010	0.000664	0.1000
1,000	0.000010	0.00100	0.010000	1.0
10,000	0.000013	0.01000	0.132900	1.7 min
100,000	0.000017	0.10000	1.661000	2.78 hr
1,000,000	0.000020	1.0	19.9	11.6 day
1,000,000,000	0.000030	16.7 min	18.3 hr	318 centuries

Getting in front

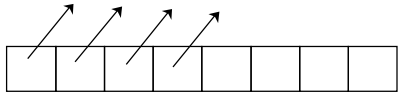
- Suppose we want to add a new element
  - At the back of a string or an ArrayList or a …
  - At the front of a string or an ArrayList or a …
  - Is there a difference? Why? What's complexity?
- Suppose this is an important problem: we want to grow at the front (and perhaps at the back)
  - Think editing film clips and film splicing
  - Think DNA and gene splicing
- Self-referential data structures to the rescue
  - References, reference problems, recursion, binky

What's the Difference Here?

- How does find-a-track work? Fast forward?

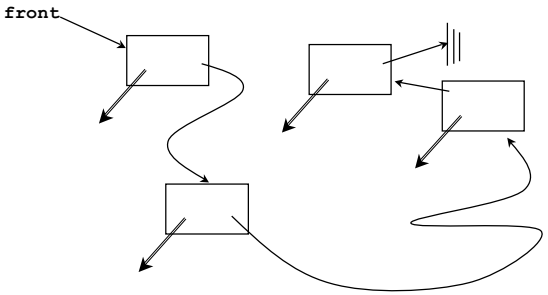
Contrast LinkedList and ArrayList

- See ISimpleList, SimpleLinkedList, SimpleArrayList
  - Meant to illustrate concepts, not industrial-strength
  - Very similar to industrial-strength, however
- ArrayList --- why is access O(1) or constant time?
  - Storage in memory is contiguous, all elements same size
  - Where is the 1^{st} element? 40^{th}? 360^{th}?
  - Doesn't matter what's in the ArrayList, everything is a pointer or a reference (what about null?)



What about LinkedList?

- Why is access of N^{th} element linear time?
- Why is adding to front constant-time O(1)?



ArrayLists and linked lists as ADTs

- As an ADT (abstract data type) ArrayLists support
  - *Constant-time* or O(1) access to the k-th element
  - *Amortized* linear or O(n) storage/time with `add`
    - Total storage used in n-element vector is approx. 2n, spread over all accesses/additions (why?)
  - Adding a new value in the middle of an ArrayList is expensive, linear or O(n) because shifting required
- Linked lists as ADT
  - Constant-time or O(1) insertion/deletion anywhere, but…
  - Linear or O(n) time to find where, sequential search
- Good for *sparse* structures: when data are scarce, allocate exactly as many list elements as needed, no wasted space/copying (e.g., what happens when vector grows?)

Linked list applications

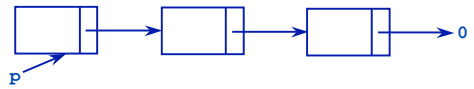
- Remove element from middle of a collection, maintain order, no shifting. Add an element in the middle, no shifting
  - What's the problem with a vector (array)?
  - Emacs visits several files, internally keeps a linked-list of *buffers*
  - Naively keep characters in a linked list, but in practice too much storage, need more esoteric data structures
- What's $(3x^5 + 2x^3 + x + 5) + (2x^4 + 5x^3 + x^2 + 4x)$?
  - As a vector `(3, 0, 2, 0, 1, 5)` and `(0, 2, 5, 1, 4, 0)`
  - As a list `((3,5), (2,3), (1,1), (5,0))` and _____?
  - Most polynomial operations sequentially visit terms, don't need random access, do need "splicing"
- What about $(3x^{100} + 5)$?

Linked list applications continued

- If programming in C, there are no "growable-arrays", so typically linked lists used when # elements in a collection varies, isn't known, can't be fixed at compile time
  - Could grow array, potentially expensive/wasteful especially if # elements is small.
  - Also need # elements in array, requires extra parameter
  - With linked list, one pointer used to access all the elements in a collection
- Simulation/modeling of DNA gene-splicing
  - Given list of millions of CGTA… for DNA strand, find locations where new DNA/gene can be spliced in
    - Remove target sequence, insert new sequence

Linked lists, CDT and ADT

- As an ADT
  - A list is empty, or contains an element and a list
  - `( )` or `(x, (y, ( ) ) )`
- As a picture



- As a CDT (concrete data type)

```
public class Node
{
     String value;
     Node next;
};
```

```
Node p = new Node();
p.value = "hello";
p.next = null;
```


Building linked lists

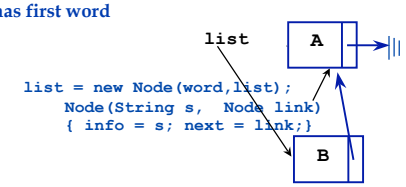
- Add words to the front of a list (draw a picture)
  - Create new node with next pointing to list, reset start of list

```
public class Node {
   String value;
   Node next;
   Node(String s, Node link){
     value = s;
     next = link;
   }
};
// … declarations here
Node list = null;
while (scanner.hasNext()) {
    list = new Node(scanner.next(), list);
}
```

- What about adding to the end of the list?

Dissection of add-to-front

- List initially empty
- First node has first word



- Each new word causes new node to be created
  - New node added to front
- Rhs of operator = completely evaluated before assignment

Standard list processing (iterative)

- Visit all nodes once, e.g., count them or *process* them

```
public int size(Node list){
     int count = 0;
     while (list != null) {
        count++;
        list = list.next;
     }
     return count;
}
```

- What changes in code above if we change what "process" means?
  - Print nodes?
  - Append "s" to all strings in list?

Nancy Leveson: Software Safety

Founded the field

- Mathematical and engineering aspects
  - Air traffic control
  - Microsoft word

"C++ is not state-of-the-art, it's only state-of-the-practice, which in recent years has been going backwards"

- Software and steam engines: once extremely dangerous?
  - http://sunnyday.mit.edu/steam.pdf
- THERAC 25: Radiation machine that killed many people
  - http://sunnyday.mit.edu/papers/therac.pdf

Building linked lists continued

- What about adding a node to the end of the list?
  - Can we search and find the end?
  - If we do this every time, what's complexity of building an N-node list? Why?
- Alternatively, keep pointers to first and last nodes of list
  - If we add node to end, which pointer changes?
  - What about initially empty list: values of pointers?
    - Will lead to consideration of header node to avoid special cases in writing code
- What about keeping list in order, adding nodes by splicing into list? Issues in writing code? When do we stop searching?

Standard list processing (recursive)

- Visit all nodes once, e.g., count them

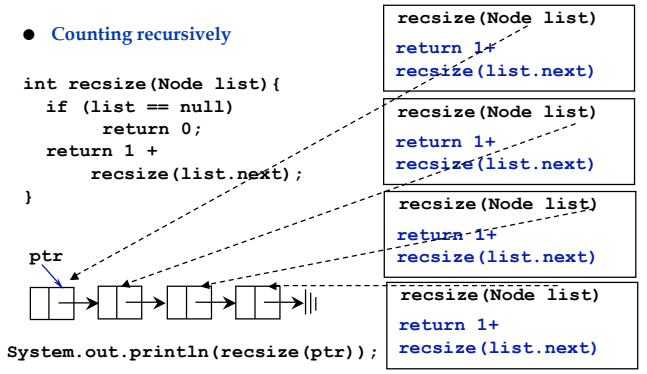
```
public int recsize(Node  list) {
    if (list == null) return 0;
    return 1 + recsize(list.next);
}
```

- Base case is almost always empty list: null pointer
  - Must return correct value, perform correct action
  - Recursive calls use this value/state to anchor recursion
  - Sometimes one node list also used, two "base" cases
- Recursive calls make progress towards base case
  - Almost always using `list.next` as argument

Recursion with pictures

- Counting recursively

```
int recsize(Node list){
  if (list == null)
       return 0;
  return 1 +
       recsize(list.next);
}
```



Recursion and linked lists

- Print nodes in reverse order
  - Print all but first node and…
    - Print first node before or after other printing?

```
public void print(Node list) {
    if (list != null) {
       System.out.println(list.info);
       print(list.next);
    }
}
```


Complexity Practice

- What is complexity of *Build*? (what does it do?)

```
public Node build(int n) {
    if (null == n) return null;
    Node first = new Node(n, build(n-1));
    for(int k = 0; k < n-1; k++) {
        first = new Node(n,first);
    }
    return first;
}
```

- Write an expression for T(n) and for T(0), solve.
  - Let T(n) be time for build to execute with n-node list
  - T(n) = T(n-1) + O(n)

Changing a linked list recursively

- Pass list to method, return altered list, assign to list
  - Idiom for changing value parameters

```
list = change(list, "apple");
public Node change(Node list, String key) {
  if (list != null) {
     list.next = change(list.next, key);
     if (list.info.equals(key)) return list.next;
     else                       return list;
  }
  return null;
 }
```

- What does this code do? How can we reason about it?
  - Empty list, one-node list, two-node list, *n*-node list
  - Similar to proof by induction