CPS 130: Homework I (Due September 12)

1 Asymptotic notation

1. Obtain asymptotic upper and lower bounds for:
   (a) $20n^5 - 1000n^4 + 10n$.
   (b) $a^{n-1} + a^{n-2} + \ldots + a + 1$.

2. Sort the following functions by their asymptotic order (see CLRS problem 3-3a for more related problems).
   $\lg(\lg^* n)$, $\lg^2 n$, $\ln \ln n$, $\lg^* n$, $\sqrt{\lg n}$

3. Solve the CLRS Problem 3-1.

2 Sums

1. Evaluate the following sums.
   (a) $\sum_{i=0}^{n} i \binom{n}{i}$
   (b) $\sum_{i=1}^{n} 2^{n-i} i^2$

3 Recursion

1. Evaluate the following recurrences.
   (a) $T(n) = 3T(n/2) + n\lg n$
   (b) $T(n) = T(n-1) + 1/n$
   (c) $T(n) = \sqrt{n}T(\sqrt{n}) + n$
   (d) $T(n) = 4T(n/2) + n^2 \sqrt{n}$

2. Solve CLRS Exercise 4.2-2 (on page 72). Also verify your answer by substitution method.

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1CLRS refers to the “Intro to Algorithms” book (2nd edition), by Cormen et al.
3. Solve CLRS Ex. 4.3-1 (part a only) using both the recursion tree and master theorem. Also verify your answers using substitution method.

4. Solve CLRS Ex. 4.3-4