CPS 130: Homework II (Due September 26)

1. Sorting and searching

1. Show that the second smallest of $n$-elements can be found with $n + \lfloor \log(n) \rfloor - 2$ comparisons.

2. Describe an algorithm that, given $n$ integers preprocesses its input and then answers any query about how many of the $n$ integers fall into a range $[a \ldots b]$ in $O(1)$ time. Your algorithm should use $\Theta(n + k)$ preprocessing time.

2. Heaps and heapsort

1. What are the minimum and maximum numbers of elements in a heap of height $h$?

2. Show that an $n$-element heap has height $\lfloor \log n \rfloor$.

3. Show that the worst-case running time of MAX-HEAPIFY on a heap of size $n$ is $\Omega(\log n)$.

4. Do (CLRS-Problem 6.2) on $d$-ary heaps.

3. Quicksort

1. Suppose that the splits at every level of quicksort are in proportion $1 - \alpha$ to $\alpha$, where $0 < \alpha \leq 1/2$ is a constant. Show that the minimum depth of a leaf in the recursion tree is approximately $-\log n / \log \alpha$ and the maximum depth is approximately $-\log n / \log(1 - \alpha)$.