## Logic Intro

CPS 270 Ron Parr

### Historical Perspective I

- · Logic was one of the classical foundations of AI
- Dream: A Knowledge-Based agent
  - Tell the agent facts
  - Agent uses rules of inference to deduce consequences
     Example: prolog
- Distinction between data and program
- Embodied in field of "Expert Systems"

### Example: Minesweeper

- How do you play minesweeper?
- How would you program a machine to do it?
   Hacking
  - Search
  - Logic
- · Logic approach
  - Tell the system of rules of minesweeper
  - System uses logic to make the best moves

## What is logic, really?

- Syntax: Rules for constructing valid sentences
- Semantics: Relate syntax to the real world

### Entailment

- Aim: Rule for generating (or testing) new sentences that are *necessarily* true
- The truth of sentence may depend upon the *interpretation* of the sentence

### Interpretations

- An interpretation is a way of matching up objects in the universe with symbols in a sentence (or database).
- A sentence may be true in one interpretation, but false in another
- A *necessarily true* sentence is true in all interpretations

### Soundness & Completeness

- A (set of) rule(s) of inference is sound if it generates only sentences that are entailed by the knowledge base, i.e., only necessary truths
- A (set of) rule(s) of inference is complete if it can generate all necessary truths
- Can we have one w/o the other?

#### Historical Perspective II

- Things that are not true necessarily but still true are sometimes said to be "contingent," "accidental," or "synthetic," truths.
- A deep understanding of this distinction evolved through thousands of years of philosophy and mathematics
- Arguably one of the most important intellectual accomplishments of mankind
  - Basis of mathematic proofs
  - Provides a rigorous procedure for verifying statements

### Relation to SAT

- When we want to know if a sentence is satisfiable, what does this mean?
- What about #SAT?
- Why do we care?

### **Propositional Logic**

- Propositional logic is the simplest logic
- All sentences are composed of – Atoms
  - Negation
  - Disjunction, conjunction
  - Conditional, biconditionals
- Atoms can map to any proposition about the universe

## **Checking Validity**

- Classic method for checking validity: truth table
- Enumerate all possible values (t/f) of atomic elements of a sentence

$$(P \lor H) \land \neg H \Longrightarrow P$$

• Enumerate all 4 (or more) combinations

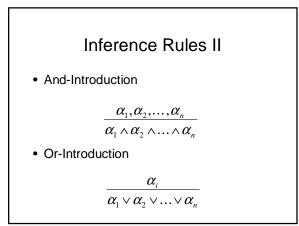
## Inference Rules

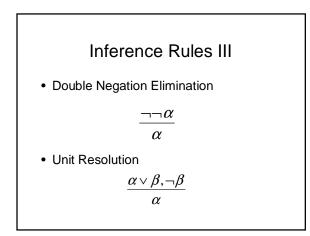
- Inference rules are (typically) sound methods of generating new sentences given a set of previous sentences
- Inference rules save us the trouble of generating truth tables all of the time

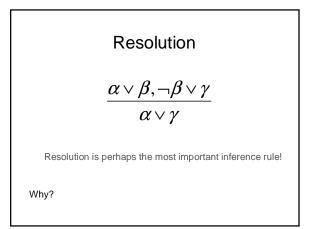
Inference Rules I  
• Modus Ponens  

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$
• And-Elimination  

$$\frac{\alpha_1 \land \alpha_2 \land \dots \land \alpha_n}{\alpha_i}$$

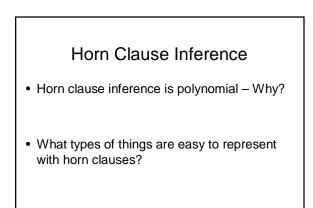






### Complexity of Inference

- What is the complexity of verifying the validity of a sentence with n literals (variables)?
- Special Case: Horn Logic
  - Horn clauses are disjunctions with at most one positive literal
  - Equivalent to



# Propositional Logic Conclusion

- Logic gives formal rules for reasoning
- Necessarily true = true in all interpretations
- Satisfiable = true in some, but not necessarily all interpretations
- Sound inference rules generate necessary truths given a KB
- Resolution is a sound and complete inference rule
- Inference with a horn KB is poly time