Reinforcement Learning (Lecture 2)

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CPS 270

RL Highlights

- Everybody likes to learn from experience
- Use ML techniques to generalize from *relatively small amounts* of experience
- Some notable successes:
  - Backgammon
  - Flying a helicopter upside down

From Andrew Ng's home page

- Sutton's seminal RL paper is 42nd most cited paper in computer science (Citeseer 10/05)

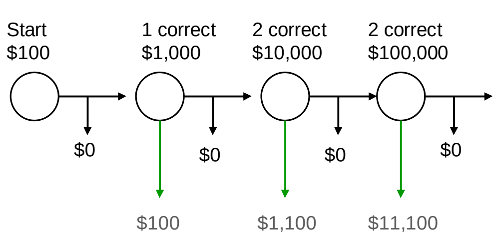
Comparison w/Other Kinds of Learning

- Learning often viewed as:
  - Classification (supervised), or
  - Model learning (unsupervised)
- RL is between these (delayed signal)
- What the last thing that happens before an accident?

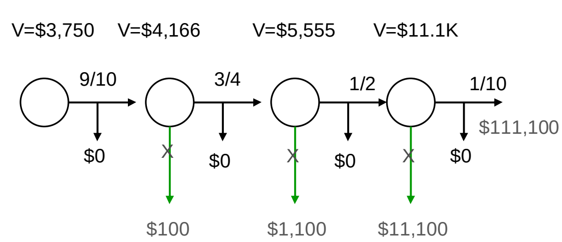
Overview

- Review of value determination
- Motivation for RL
- Algorithms for RL
  - Overview
  - TD
  - Q-learning
  - Approximation

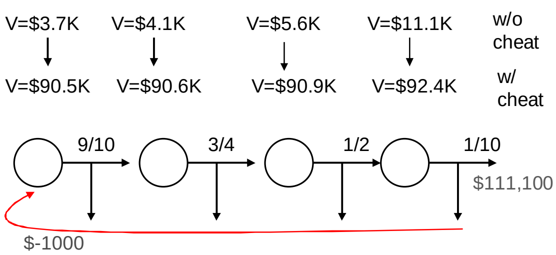
Recall Our Game Show



Optimal Policy w/o Cheating



Cheat until you win policy



Solving for Values

$$\mathbf{V} = \gamma \mathbf{P}_\pi \mathbf{V} + \mathbf{R}$$

For moderate numbers of states we can solve this system exacty:

$$\mathbf{V} = \underbrace{(\mathbf{I} - \gamma \mathbf{P}_\pi)^{-1}}\mathbf{R}$$

Guaranteed invertible because $\gamma \mathbf{P}_\pi$ has spectral radius <1

Iteratively Solving for Values

$$\mathbf{V} = \gamma \mathbf{P}_\pi \mathbf{V} + \mathbf{R}$$

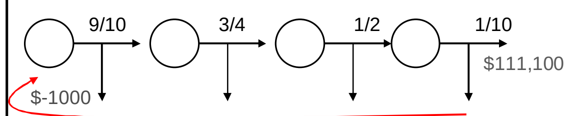
For larger numbers of states we can solve this system indirectly:

$$\mathbf{V}^{i+1} = \gamma \mathbf{P}_\pi \mathbf{V}^{i} + \mathbf{R}$$

Guaranteed convergent because $\gamma \mathbf{P}_\pi$ has spectral radius <1 for γ<1

Convergence not guaranteed for γ=1

Iterative Policy Evaluation



0.00	0.00	0.00	0.00
-100.00	-250.00	-500.00	10210.00
-335.00	-650.00	4555.00	10120.00
-718.50	3082.50	4392.50	9908.50
2602.40	2864.75	4095.00	9563.35
2738.52	3471.85	5582.88	12552.16

Iterations ↓

Iterations Contd.

i=0	0.00	0.00	0.00	0.00
i=1	-100.00	-250.00	-500.00	10210.00
i=2	-335.00	-650.00	4555.00	10120.00
i=3	-718.50	3082.50	4392.50	9908.50
i=4	2602.40	2864.75	4095.00	9563.35
i=5	2738.52	3471.85	5582.88	12552.16
i=20	15697.49	16688.07	18396.47	23621.43
i=100	56740.99	57190.86	58074.31	60999.20
i=200	74658.96	74872.93	75399.39	77318.76
i=1000	82469.80	82580.93	82951.31	84432.82
i=10000	82470.37	82581.48	82951.85	84433.33

Note: Slow convergence b/c γ=1

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Why We Need RL

- Where do we get transition probabilities?
- How do we store them?
  - Big problems have big models
  - Model size is quadratic in state space size
- Where do we get the reward function?

RL Framework

- Learn by “trial and error”
- No assumptions about model
- No assumptions about reward function
- Assumes:
  - True state is known at all times
  - Immediate reward is known
  - Discount is known

RL Schema

- Act

- Perceive results

- Update something

- Repeat

RL for Our Game Show

- Problem: Don’t know prob of answering correctly
- Solution:
  - Buy the home version of the game
  - Practice on the home game to refine our strategy
  - Deploy strategy when we play the real game

Model Learning Approach

- Learn model, solve
- How to learn a model:
  - Take action a in state s, observe s’
  - Take action a in state s, n times
  - Observe s’ m times
  - P(s’|s,a) = m/n
  - Fill in transition matrix for each action
  - Compute avg. reward for each state
- Solve learned model as an MDP

Limitations of Model Learning

- Partitions learning, solution into two phases
- Model may be large (hard to visit every state lots of times)
  - Note: Can’t completely get around this problem…
- Model storage is expensive
- Model manipulation is expensive

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Temporal Difference Learning

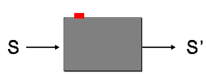
- One of the first RL algorithms
- Learn the value of a *fixed* policy (no optimization; just prediction)
- Recall iterative value determination:

$$V^{i+1}(s) = R(s,\pi(s)) + \gamma \sum_{s'} P(s'|s,\pi(s)) V^i(s')$$

Problem: We don't know this.

First Idea: Monte Carlo Sampling

- Assume that we have a black box:



- Count the number of times we see each s'
  - Estimate P(s'|s) for each s'
  - Essentially learns a mini-model for state s
  - Can think of as numerical integration
- Problem: The world doesn't work this way

Next Idea

- Remember Value Determination:

$$V^{i+1}(s) = R(s,\pi(s)) + \gamma \sum_{s'} P(s'|s,\pi(s)) V^i(s')$$

- Compute an update *as if the observed s' and r were the only possible outcomes*:

$$V^{temp}(s) = r + \gamma V^i(s')$$

- Make a small update in this direction:

$$V^{i+1}(s) = (1-\alpha)V^i(s) + \alpha V^{temp}(s)$$

$$0 < \alpha \le 1$$

Idea: Value Function Soup

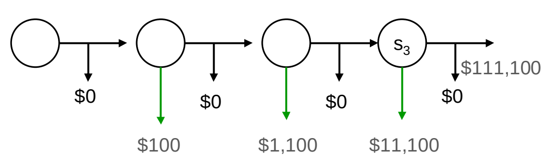
Suppose: $\alpha = 0.1$

Upon observing s':
•Discard 10% of soup
•Refill with $V^{temp}(s)$
•Stir
•Repeat

One vat for each state

$$V^{i+1}(s) = (1-\alpha)V^i(s) + \alpha V^{temp}(s)$$

Example: Home Version of Game



Suppose we guess: $V(s_3)$=15K
We play and get the question wrong

$V^{temp}=0$
$V(s_3) = (1-\alpha)15K + \alpha 0$

Convergence?

- Why doesn't this oscillate?
  - e.g. consider some low probability s' with a very high (or low) reward value

  - This could still cause a big jump in V(s)

Convergence Intuitions

- Need heavy machinery from stochastic process theory to prove convergence
- Main ideas:
  - Iterative value determination converges
  - Updates approximate value determination
  - Samples approximate expectation

$$V^{i+1}(s) = R(s,\pi(s)) + \gamma \sum_{s'} P(s'|s,\pi(s)) V^i(s')$$

Ensuring Convergence

- Rewards have bounded variance
- $0 \le \gamma < 1$
- Every state visited infinitely often
- Learning rate decays so that:
  - $\sum_i^\infty \alpha_i(s) = \infty$
  - $\sum_i^\infty \alpha_i^2(s) < \infty$

These conditions are jointly *sufficient* to ensure convergence in the limit with probability 1.

How Strong is This?

- Bounded variance of rewards: easy
- Discount: standard
- Visiting every state infinitely often: Hmmm…
- Learning rate: Often leads to slow learning
- Convergence in the limit: Weak
  - Hard to say anything stronger w/o knowing the mixing rate of the process
  - Mixing rate can be low; hard to know a priori
- Convergence w.p. 1: Not a problem.

Using TD for Control

- Recall value iteration:

$$V^{i+1}(s) = \max_a R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^i(s')$$

- Why not pick the maximizing **a** and then do:

$$V^{i+1}(s) = (1-\alpha)V^i(s') + \alpha V^{temp}(s')$$

  - s' is the observed next state after taking action **a**

Problems

- Pick the best action w/o model?
- Must visit every state infinitely often
  - What if a good policy doesn't do this?
- Learning is done "on policy"
  - Taking random actions to make sure that all states are visited will cause problems

Q-Learning Overview

- Want to maintain good properties of TD
- Learns good policies and optimal value function, not just the value of a fixed policy
- Simple modification to TD that learns the optimal policy regardless of how you act! (mostly)

Q-learning

- Recall value iteration:

$$V^{i+1}(s) = \max_a R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^i(s')$$

- Can split this into two functions:

$$Q^{i+1}(s,a) = R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^i(s')$$

$$V^{i+1}(s) = \max_a Q^{t+1}(s,a)$$

Q-learning

- Store Q values instead of a value function
- Makes selection of best action easy
- Update rule:

$$Q^{temp}(s,a) = r + \gamma \max_{a'} Q^i(s',a')$$

$$Q^{i+1}(s,a) = (1-\alpha) Q^i(s,a) + \alpha Q^{temp}(s,a)$$

Q-learning Properties

- Converges under same conditions as TD
- Still must visit every state infinitely often
- Separates policy you are currently following from value function learning:

$$Q^{temp}(s,a) = r + \gamma \max_{a'} Q^i(s',a')$$

$$Q^{i+1}(s,a) = (1-\alpha) Q^i(s,a) + \alpha Q^{temp}(s,a)$$

Value Function Representation

- Fundamental problem remains unsolved:
  - TD/Q learning solves model-learning problem, but
  - Large models still have large value functions
  - Too expensive to store these functions
  - Impossible to visit every state in large models
- Function approximation
  - Use machine learning methods to generalize
  - Avoid the need to visit every state

Function Approximation

- General problem: Learn function f(s)
  - Linear regression
  - Perceptron (single layer neural network)
  - Neural networks
- Idea: Approximate f(s) with g(s,θ)
  - g is some easily computable function of s and θ
  - Try to find θ that minimizes the error in g

Linear Regression

- Define a set of basis functions (vectors)

$$h_1(s), h_2(s)\ldots h_k(s)$$

- Approximate f with a weighted combination of these

$$g(s) = \sum_{j=1}^{k} \theta_j h_j(s)$$

- Example: Space of quadratic functions:

$$h_1(s) = 1, h_2(s) = s, h_3(s) = s^2$$

- Orthogonal projection minimizes SSE

Updates with Approximation

- Recall regular TD update:

$$V^{i+1}(s) = (1-\alpha)V^i(s) + \alpha V^{temp}(s)$$

- With function approximation:

$$V(s) \approx V(s,\theta)$$

- Update:

$$\theta^{i+1} = \theta^i + \alpha(V^{temp} - V(s,\theta))\nabla_\theta V(s,\theta)$$

Vector operations

For linear value functions

- Gradient is trivial:

$$V(s,\theta) = \sum_{j=1}^{k} \theta_j h_j(s)$$

$$\nabla_{\theta_j} V(s,\theta) = h_j(s)$$

- Update is trivial:

$$\theta_j^{i+1} = \theta_j^{i} + \alpha(V^{temp} - V(s,\theta))h_j(s)$$

Individual components

Neural Networks

- s = input into neural network
- w = weights of neural network
- g(s, θ) = output of network
- Try to minimize

$$E = \sum_s \left(f(s) - g(s,\theta)\right)^2$$

- Compute gradient of error WRT weights

$$\frac{\partial E}{\partial \theta}$$

- Adjust to minimize error

Combining NNs with TD

- Recall TD:

$$V^{temp}(s) = R(s) + \gamma V^i(s')$$

$$V^{i+1}(s) = (1-\alpha)V^i(s) + \alpha V^{temp}(s)$$

- Compute error function:

$$E = \left(V^i(s,w) - V^{temp}(s,\theta)\right)^2$$

- Update:

$$\theta^{i+1} = \theta^i - \alpha \frac{\partial E}{\partial \theta}$$

$$= \theta^i + 2\alpha\left[V^{temp}(s,\theta) - V(s,\theta)\right]\frac{\partial V(s,\theta)}{\partial \theta}$$

Gradient-based Updates

$$\theta^{i+1} = \theta^i - \alpha \frac{\partial E}{\partial \theta}$$

$$= \theta^i + 2\alpha\left[V^{temp}(s,\theta) - \hat{V}(s,\theta)\right]\frac{\partial V(s,\theta)}{\partial \theta}$$

- Equivalent to one step of backprop with V^{temp} as target
- Constant factor absorbed into learning rate
- Table-updates are a special case
- Perceptron, linear regression are special cases

Properties of approximate RL

- Table-updates are a special case
- Can be combined with Q-learning
- Convergence not guaranteed
  - Function approximators typically converge to local optimum
  - Convergence ***NOT*** guaranteed when combined with RL
    - Chasing a moving target
    - Errors can compound
- Success requires very well chosen features

Other Approaches

- TD, Q-learning approximate value iteration
- Typically use parameterized V
- Can also approximate policy iteration
  - Parameterized space of policies
  - Estimate values from samples
  - Update policy parameters to improve performance

How'd They Do That???

- Helicopter (Ng et al.)
  - Approximate policy iteration
  - Constrained policy space
  - Trained on a simulator
- Backgammon (Tesauro)
  - Predecessor: Neuro-Gammon
  - Generalize RL to alternating move games (already done by Samuel)
  - Neural network value function approximation
  - Used TD
    - Model was known
    - Action space was large
    - Exploration/On policy evaluation?
  - Carefully selected inputs to neural network
  - About 1.5 million games played against self

Swept under the rug…

- Difficulty of finding good features
- Partial observability
- Exploration vs. Exploitation

Conclusions

- Reinforcement learning solves an MDP
- Converges for exact value function representation
- Can be combined with approximation methods
- Good results require good features