CPS 270
Search I

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What is Search?

- Search is a basic problem-solving method
- We start in an initial state
- We examine states that are (usually) connected by a sequence of actions to the initial state
- We aim to find a solution, which is a sequence of actions that brings us from the initial state to the goal state, minimizing cost

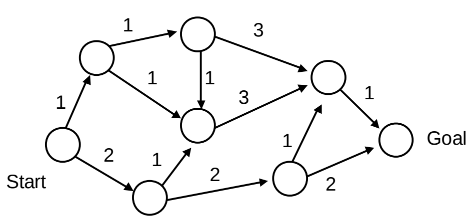
Overview

- Problem Formulation
- Uninformed Search
  – DFS, BFS, IDDFS, etc.
- Informed Search
  – Greedy, A*
- Properties of Heuristics

Problem Formulation

- Four components of a search problem
  – Initial State
  – Actions
  – Goal Test
  – Path Cost
- Optimal solution = lowest path cost to goal

Example: Path Planning



Find shortest route from one city to another using highways.

Example 8(15)-puzzle

8	3	4
1		7
2	6	5

Possible Start State

Solution →

	1	2
3	4	5
6	7	8

Goal State

Actions: UP, DOWN, RIGHT, LEFT

“Real” Problems

- Robot motion planning
- Drug design
- Logistics
  - Route planning
  - Tour Planning
- Assembly sequencing
- Internet routing

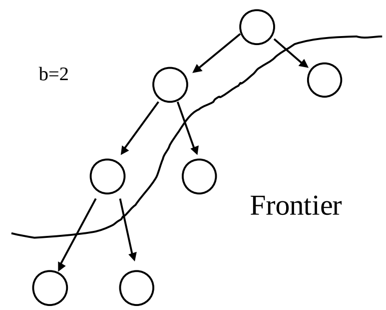
Why Use Search?

- Other algorithms exist for these problems:
  - Dijkstra’s Algorithm
  - Dynamic programming
  - All-pairs shortest path
- Use search when it is too expensive to enumerate all states
- 8-puzzle has 362,800 states
- 15-puzzle has 1.3 trillion states
- 24-puzzle has 10^{25} states

Basic Search Concepts

- Assume a tree-structured space (for now)
- Nodes: Places in search tree
  (states exist in the problem space)
- Search tree: portion of state space visited so far
- Expansion: Generation of successors for a state
- Frontier: Set of states visited, but not expanded
- Branching factor: Max no. of successors = b
- Goal depth: Depth of shallowest goal = d

Example Search Tree



Generic Search Algorithm

```
Function Tree-Search(problem, Queuing-Fn)

    fringe = Make-Queue(Make-Node(Initial-State(problem)))
    loop do
        if empty(fringe) then return failure
        node = pop(fringe)
        if Goal-Test(problem, state) then return node
        fringe = Add-To-Queue(fringe, expand(node, problem))
    end
```

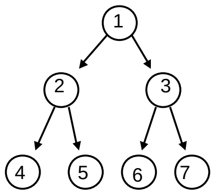
Interesting details are in the implementation of Add-To-Queue

Evaluating Search Algorithms

- Completeness:
  - Is the algorithm guaranteed to find a solution when there is one?
- Optimality:
  - Does the algorithm find the optimal solution?
- Time complexity
- Space complexity

Uninformed Search: BFS

Frontier is a FIFO

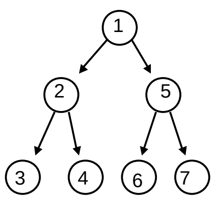


BFS Properties

- Completeness: Y
- Optimality: Y (for uniform cost)
- Time complexity: $O(b^{d+1})$
- Space complexity: $O(b^{d+1})$

Uninformed Search: DFS

Frontier is a LIFO



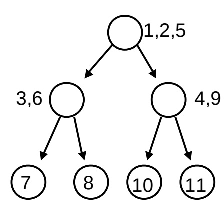
DFS Properties

- Completeness: N (unless tree is finite)
- Optimality: N
- Time complexity: $O(b^m)$ (m = depth we hit, m>d?)
- Space complexity: O(bm)

Iterative Deepening

- Want:
  – DFS memory requirements
  – BFS optimality, completeness
- Idea:
  – Do a depth-limited DFS for depth m
  – Iterate over m

IDDFS



IDDFS Properties

- Completeness: Y
- Optimality: Y (whenever BFS is optimal)
- Time complexity: $O(b^{d+2})$
- Space complexity: O(bd)

IDDFS vs. BFS

Theorem: IDDFS visits no more than twice as many nodes for a binary tree as BFS.

Proof: Assume the tree bottoms out at depth d, BFS visits:

$$2^{d+1}-1$$

In the worst case, IDDFS does no more than:

$$\sum_{i=0}^{d}(2^{i+1}-1)=\sum_{i=0}^{d}2^{i+1}-\sum_{i=0}^{d}1=(2^{d+2}-1)-(d+1)<2(2^{d+1}-1)$$

What about b-ary trees? IDDFS relative cost is lower!

Bi-directional Search

$$b^{d/2}+b^{d/2}<<b^d$$

Issues with Bi-directional Search

- Uniqueness of goal
  - Suppose goal is parking your car
  - Huge no. of possible goal states (configurations of other vehicles)
- Invertability of actions

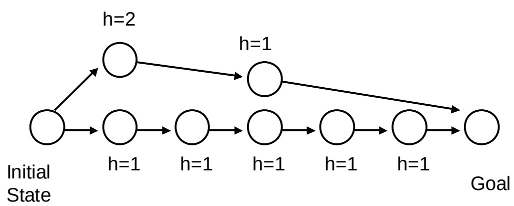
Informed Search

- Idea: Give the search algorithm hints
- Heuristic function: h(x)
- h(x) = estimate of cost to goal from x
- If h(x) is 100% accurate, then we can find the goal in O(bd) time

Greedy Search

- Expand node with lowest h(x)
- Optimal if h(x) is 100% correct
- How can we get into trouble with this?

What Price Greed?



What's broken with greedy search?

A*

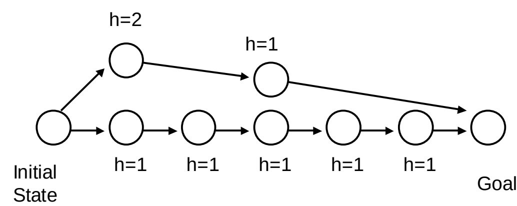
- Path cost so far: g(x)
- Total cost estimate: f(x) = g(x) + h(x)
- Maintain frontier as a priority queue
- O(bd) time if h is 100% accurate
- We want h to be an *admissable* heuristic
- Admissable: never overestimates cost

A* Properties

Theorem: A* is optimal if h(x) is admissable.

Proof sketch: Suppose a suboptimal goal node g2 appears on the fringe. If C* is the optimal cost, f(g2) > C*. Since h never overestimates the cost, there must exist some unexpanded node along the optimal path that has not yet been expanded. Thus, as long as we have not yet found the optimal path, we will continue to expand nodes

Does A* fix the greedy problem?



Properties of Heuristics

- h2 dominates h1 if h2(x)>h1(x) for all x
- Does this mean that h2 is better?
- Suppose you have multiple admissable heuristics. How do you combine them?

Developing Heuristics

- Is it hard to develop admissable heuristics?
- What are some heuristics for the 8 puzzle?
- What is a general strategy for developing admissable heuristics?

Other Issues

- Graphs
  - What issues arise?
  - Monotonicity
- Non-uniform costs
- Accuracy of heuristic
- A* is optimally efficient