CPS 196.2

Expressive negotiation over donations

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One donor (bidder)

\[ u(\text{money}) = 1 \]
\[ u(\text{UNICEF}) = 0.8 \]

\[ U = 1 \]
Two independent donors

\[ u(\text{donor 1}) = 1 \]
\[ u(\text{donor 2}) = 0.8 \]

\[ U = 1 \]
Two donors with a contract

\[ u( \text{donor 1}) = 1 \]
\[ u( \text{donor 2}) = .8 \]

\[ U = .5 + .8 = 1.3 > 1 \]

\[ u( \text{donor 1}) = 1 \]
\[ u( \text{donor 2}) = .8 \]

\[ U = .5 + .8 = 1.3 > 1 \]
Contracting using matching offers

I’ll match any donation to $\text{UNICEF}$.

- $u(\text{unicef, $100}) = 1$
- $u(\text{unicef, $70 and $30}}) = 1.3$
Limitations of matching offers

- One-sided
- Involve only a single charity
Two charities

\[ u(\text{charity 1}) = 1 \]
\[ u(\text{charity 2}) = .8 \]
\[ u(\text{charity 3}) = .3 \]

\[ U = 1.1 \]
A different approach

- Donors can submit bids indicating their preferences over charities

- A center accepts all the bids and decides who pays what to whom
What do we need?

- A general **bidding language** for specifying “complex matching offers” (bids)

- Algorithms for the **clearing problem** (given the bids, who pays what to whom)
One charity

• A bid for one charity:

“Given that the charity ends up receiving a total of $x$ (including my contribution), I am willing to contribute at most $w(x)$”

Bidder’s maximum payment

Budget

$w(x)$

$x = \text{total payment to charity}$
Bid 1

x = total payment

w(x)

maximum payment

Budget

$10

$100

$500

$50

$30

$10
Bid 2

$w(x)$

maximum payment

$75$

$45$

$15$

$10$

$100$

$500$

$x = \text{total payment}$

Budget
Current solution

$w(x) = \text{total payment}$

bidders are willing to make

$\text{max surplus}$

$\text{max donated}$

$x = \text{total payment}$

$\$125$

$\$75$

$\$25$

$\$10$

$\$100$

$\$43.75$

$\$500$
Tsunami event (Dagstuhl 05)
Problem with more than one charity

- Willing to give $1 for every $100 to UNICEF
- Willing to give $2 for every $100 to Amnesty Int’l
- BUDGET: $50

- Could get stuck paying $100!
- Most general solution: \( w(x_1, x_2, \ldots, x_m) \)
  - Requires specifying exponentially many values
Solution: separate utility and payment; assume utility decomposes

- Willing to give $1 for every $100 to UNICEF
- Willing to give $2 for every $100 to Amnesty Int’l
- Budget constraint: $50
The general form of a bid

\[ w(u^1(x_1) + u^2(x_2) + \ldots + u^m(x_m)) \]

\[ u^1(x_1) + u^2(x_2) + \ldots + u^m(x_m) \text{ (utils)} \]
What to do with the bids?

- Decide $x_1, x_2, \ldots, x_m$ (total payment to each charity)
- Decide $y_1, y_2, \ldots, y_n$ (total payment by each bidder)

**Say $x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n$ is valid if**

- $x_1 + x_2 + \ldots + x_m \leq y_1 + y_2 + \ldots + y_n$ (no more money given away than collected)
- For any bidder $j$, $y_j \leq w_j(u_j^1(x_1) + u_j^2(x_2) + \ldots + u_j^m(x_m))$ (nobody pays more than they wanted to)
Objective

- Among valid outcomes, find one that maximizes

- **Total donated** = $x_1 + x_2 + \ldots + x_m$

- **Surplus** = $y_1 + y_2 + \ldots + y_n - x_1 - x_2 - \ldots - x_m$
Avoiding indirect payments
No payments to disliked charities
Hardness of clearing

- NP-complete to decide if there exists a solution with objective > 0

- That means: the problem is inapproximable to any ratio (unless P=NP)
**General program formulation**

- **Maximize**
  - $x_1 + x_2 + \ldots + x_m$, OR
  - $y_1 + y_2 + \ldots + y_n - x_1 - x_2 - \ldots - x_m$

- **Subject to**
  - $y_1 + y_2 + \ldots + y_n - x_1 - x_2 - \ldots - x_m \geq 0$
  - For all $j$: $y_j \leq w_j(u_j^1 + u_j^2 + \ldots + u_j^m)$
  - For all $i, j$: $u_j^i \leq u_j^i(x_i)$
Concave piecewise linear constraints

\[ y \leq b(x) \]

\[ y \leq l_1(x) \]

\[ y \leq l_2(x) \]

\[ y \leq l_3(x) \]
Linear programming

- So, if all the bids are \textit{concave}…
  - All the $u_{j}^{i}$ are concave \textit{(utils)}
  - All the $w_{j}$ are concave

- Then the program is a linear program (solvable to optimality in polynomial time)
- Even if they are not concave, can solve as MIP