CS296.1 Final Exam Sample — Sample Solution

1. Under perspective projection, a square in the world projects to a quadrilateral in the image. Given the quadrilateral, how can you find the image projection of the center of the square?
Answer: *Perspective projection preserves co-linearity. So the center of the square projects to the intersection of the two diagonals of the quadrilateral.*

2. When is the focal distance of a camera equal to the focal length of it lens?
Answer: *When the lens is focused at infinity.*

3. What axis is the following rotation about, by how many degrees, and in what direction (clockwise or counterclockwise)?

$$R = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[Hint: Unless you can tell by inspection, find a vector that is not changed by R. That is the axis. Then pick a simple vector orthogonal to the axis, and see by how much it is rotated.]

Answer: *R is a rotation around the z axis by 30 degrees counterclockwise.*

4. What is the Rodrigues vector for the rotation in the previous question?
Answer:

$$\mathbf{r} = \frac{\pi}{6} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} .$$

5. Write the result of convolving the finite, discrete signals

$$\begin{bmatrix} 1 & 2 & 4 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} .$$

Assume that signals are extended with zeros where they are not defined. Your result should be a vector of five numbers.
Answer:

$$\begin{bmatrix} 1 & 5 & 12 & 16 & 8 \end{bmatrix}$$

6. For each of the following functions $f(x, y)$, state whether they are separable. If they are separable under some conditions, state the conditions. For separable functions, show how they separate.

(a)

$$f(x, y) = \log(xy)$$

Answer: *Not separable.*

(b)

$$f(x, y) = \log(x + y)$$

Answer: *Separable:*

$$f(x) = g(x)h(y) \quad \textit{where} \quad g(x) = \log x \quad \textit{and} \quad h(y) = \log y$$

(c)

$$f(x, y) = xy - x + 2y - 2$$

Answer: *Separable:*

$$f(x) = g(x)h(y) \quad \textit{where} \quad g(x) = x + 2 \quad \textit{and} \quad h(y) = y - 1$$

7. After running Canny's edge detector on an image, you notice that long edges are broken into short segments separated by gaps. In addition, some spurious edges appear. For each of the two thresholds (low and high) used in hysteresis thresholding, state how you would adjust the threshold (up or down) to address both problems. Assume that a setting exists for the two thresholds that produces the desired result. Explain your answer very briefly.

Answer: *The gaps in the long edges require a lower low threshold: parts of the long edge are detected, so the high threshold is low enough for these edges, but the edges are disconnected because the low threshold is too high. Lowering the low threshold will include more pixels of the long edges.*

Eliminating the spurious edges requires a higher high threshold. The high threshold should be increased only slightly, so as not to make the long edges disappear. The assumption in the problem statement ensures that this is possible.

8. The Lucas-Kanade tracker tracks features between two frames by repeatedly solving a 2×2 linear system of the form

$$A\mathbf{x} = \mathbf{d}$$

where A is a function of image derivatives, and $\mathbf{d}$ is a function of the difference between the two frames.

(a) How can a trackable feature (usually called a "corner") be defined in terms of the eigenvalues $\lambda_1 \geq \lambda_2$ of A and some "sufficiently large" threshold τ?

Answer:

$$\lambda_1, \ \lambda_2 > \tau \ .$$

(b) How can an edge be defined in terms of those eigenvalues?
Answer:

$$\lambda_1 > \tau, \quad \lambda_2 \approx 0 \ .$$

9. A spherical ball of radius 10 cm is placed with its center at distance 100 cm from the center of projection of a camera, and so as to fill the image exactly (that is, the image of the ball barely touches the middle of each of the four sides of the image). The image has 512×512 pixels, and the camera sensor has square pixels. Assuming perfect pinhole projection, what is the focal distance of the camera in pixels?
Answer: *From* $x = fX/Z$ *with* $X = 10$, $Z = 100$, *and* $x = 512/2 = 256$ *we obtain* $f = xZ/X = 256 \times 100/10 = 2560$ *pixels.*

10. Why does the epipolar constraint simplify stereo matching?
Answer: *Because it reduces the correspondence problem from searching the entire image to searching a line.*

11. Given the essential matrix E for a stereo pair, write two systems of equations whose solutions are the epipoles in the left and right camera coordinate systems.
Answer:

$$E\mathbf{x} = 0 \quad \textit{and} \quad \mathbf{x}^T E = 0 \ .$$

12. Continuing the previous question, what are the coefficients of the left epipolar line corresponding to the right image point with camera coordinates $\mathbf{p}_R$?

Answer: *The entries of the vector* $\mathbf{p}_R^T E$.

13. State two reasons why the "Sum of Squared Differences" measure for image correlation in stereo is generally not zero even at the correct disparity.
Answer: *Image noise, geometric distortion caused by the difference between the two cameras' viewpoints. Other reasons include differences in camera gain or bias, and changes of surface brightness with viewpoint.*