

Instructions: Questions may continue on the back. Please write clearly. What I cannot read, I will not grade. Typed homework is preferable. A good compromise is to type the words and write the math by hand. Show all your work in detail.

I expect all students to adhere to the the Duke Community Standard on this assignment. I will not accept your assignment unless you sign on the line below, if you intend to return this sheet, or you copy and sign the same statement on your own paper.

I have adhered to the Duke Community Standard in completing this assignment.

Signature: _____

1. (15 points) Using proof by contrapositive, prove the following statement:

“If 9 does not divide n^2 , then 3 does not divide n .”

2. In a survey of 60 people, it was found that 25 read *Newsweek*, 26 read *Time*, and 26 read *Wired*. Also, 9 read both *Newsweek* and *Wired*, 11 read both *Newsweek* and *Time*, 8 read both *Time* and *Wired*, and 8 read no magazine at all.
- (a) (5 points) Find the number of people who read all three magazines.
 - (b) (5 points) Draw a Venn diagram representing the scenario, and label each of the eight regions with the number of people in the sets N , T , and W .
 - (c) (5 points) Determine the number of people who read exactly one magazine.

3. Prove the following statements:

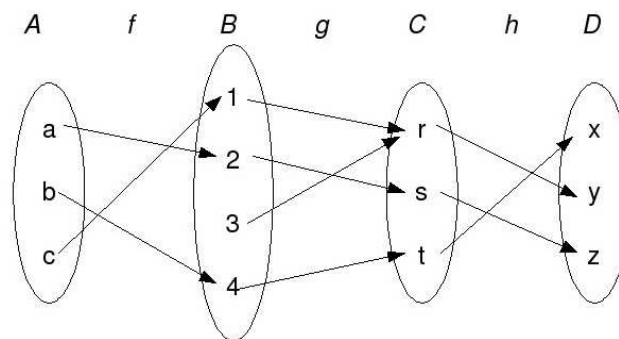
- (a) (7 points) Using proof by contradiction, prove that for all sets A and B :

$$(A - B) \cap (A \cap B) = \emptyset$$

- (b) (7 points) Using logical equivalences of sets (listed in Table 1, p 124 mostly), prove that for sets A and B :

$$A \cup (B - A) = A \cup B$$

4. What is the cardinality of each of the following sets?
 - (a) (1 point) \emptyset
 - (b) (1 point) $\{\emptyset\}$
 - (c) (1 point) $\{\emptyset, \{\emptyset\}\}$
 - (d) (1 point) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$
5. (10 points) Can you conclude that $A = B$ if A and B are two sets with the same power set? Briefly justify your answer.
6. (15 points) Rosen, 2.2.44 (p. 131). In this case “show” means give a well-reasoned explanation of the equation.
7. (15 points) Rosen, 2.3.38 (p. 147). Note that the *inverse image* of a set S is defined in just above the problem.
8. Let functions $f : A \rightarrow B, g : B \rightarrow C$, and $h : C \rightarrow D$ be defined as in the figure below.



- (a) (3 points) Determine which of the functions are one-to-one.
- (b) (3 points) Determine which of the functions are onto.
- (c) (6 points) Find the composition function $h \circ g \circ f$.