Instructions: Questions may continue on the back. Please write clearly. What I cannot read, I will not grade. Typed homework is preferable. A good compromise is to type the words and write the math by hand. Show all your work in detail.

I expect all students to adhere to the the Duke Community Standard on this assignment. I will not accept your assignment unless you sign on the line below, if you intend to return this sheet, or you copy and sign the same statement on your own paper.

I have adhered to the Duke Community Standard in completing this assignment.

Signature: ______________________________

1. (2 points) Find the prime factorization of 660.
2. (3 points) Find the gcd(2730,570) using the Euclidean algorithm.
3. Use proof by induction to prove the following results:
   (a) (15 points) $1 + 3 + 5 + ... + 2n - 1 = n^2$
   (b) (15 points) $1 + 8 + 27 + ... + n^3 = \frac{n^2(n+1)^2}{4}$
   (c) (15 points) $9^n + 3$ is divisible by 4, where n is a natural number.
4. Consider the following sequence of sums:

   $\frac{1}{1*2}$

   $\frac{1}{1*2} + \frac{1}{2*3}$

   $\frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4}$

   (a) (5 points) Calculate the sums and generalize the pattern.
   (b) (15 points) Prove your conjecture using proof by induction.

5. (10 points) Describe the error in the following "proof" via strong induction.

   All Fibonacci numbers are even.

   Remember that all Fibonacci numbers are denoted F_0, F_1, $F_2, ...$ where $F_0 = 0$, $F_1 = 1$ and $F_i = F_{i-1} + F_{i-2}$ for $i \geq 2$. The first few Fibonacci numbers are 0,1,1,2,3,5,8,13,...

   Proof: Let $P(n)$ be the statement that F_n is even. In the base case, $P(0)$ is true because $F_0 = 0$, which is even. In the inductive hypothesis, $n \geq 0$, assume that $F_0, F_1, ..., F_n$ are all even. We need to prove F_{n+1} is even. By definition, $F_{n+1} = F_n + F_{n-1}$. Since both F_n and F_{n-1} are even and the sum of two even numbers is even, then F_{n+1} is even.

6. (20 points) Prove via strong induction the **Binary Representation Theorem**: every positive integer n can be expressed as $n = c_r 2^r + c_{r-1} 2^{r-1} + ... + c_2 2^2 + c_1 2 + c_0$, where $c_r, c_{r-1}, ...c_0$ are either 0 or 1. For example $2 = 1 * 2^1 + 0$, $7 = 1 * 2^2 + 1 * 2^1 + 1$, and $6 = 1 * 2^2 + 1 * 2^1 + 0$.