

**Instructions:** Questions may continue on the back. Please write clearly. What I cannot read, I will not grade. Typed homework is preferable. A good compromise is to type the words and write the math by hand. Show all your work in detail.

**I expect all students to adhere to the the Duke Community Standard on this assignment. I will not accept your assignment unless you sign on the line below, if you intend to return this sheet, or you copy and sign the same statement on your own paper.**

*I have adhered to the Duke Community Standard in completing this assignment.*

Signature: \_\_\_\_\_

- (10 points) Barry the Burglar is trying to open a combination lock and steal lots of money from a bank vault. He knows that the NotSoSafe<sup>TM</sup> company manufactures a peculiar type of safe. He knows the lock requires 8 different integers ranging from 1 to 50 to be entered in any order. However, the lock allows for 10 different integers from the same range to be used; that is, the lock will open if any 8 of the 10 numbers in the combination are entered. Find the probability that Barry will open the vault on the first try.
- Every day, twenty distinct cars park in the same lot. Ten of these cars are compact cars and ten are luxury cars. There are only 20 spaces in this lot and all the spaces are in a row, so the cars park in a line. The position of the cars in the line will differ each day since the drivers keep different schedules.
  - (5 points) In how many different ways can the cars line up?
  - (5 points) What is the probability that on a given day, the cars alternate (e.g., compact, luxury, compact, etc)?
- Jim has a strange pair of 4-sided dice. When he rolls them, the probability of any particular outcome is proportional to the product of the outcome of each dice. All outcomes that result in a particular product are equally likely.
  - (5 points) What is the probability of the product being even?
  - (5 points) What is the probability of Jim rolling a 2 and a 3?
- Terry the Tightrope walker is drunk. He somehow manages to keep his balance, but takes a step back with probability  $1 - p$  and a step forward with probability  $p$ .
  - (10 points) What is the probability that after 2 steps on the tightrope Terry will be at the same place on the rope?
  - (10 points) What is the probability that after 3 steps Terry will be one step forward from where he began?

- (c) (10 points) Given that after 3 steps, Terry has managed to move ahead one step, what is the probability that the first step he took was a step forward?
5. Anne and Rose play a sudden death chess match. Each game ends with either a win by Anne (this happens with probability  $a$ ), a win by Rose (this happens with probability  $r$ ), or a tie (this happens with probability  $1 - a - r$ ). The match continues until one of the players wins a game (and the match).
- (a) (10 points) What is the probability that Anne will win the last game of the match?
- (b) (10 points) Given that a match lasted no more than 5 games, what is the probability that Anne won in the first game?
- (c) (10 points) Given that the match lasted no more than 5 games, what is the probability that Anne won the match?
- (d) (10 points) Given that Anne won the match, what is the probability that she won at or before the 5th game?