

**Due Date: October 6, 2008**

**Problem 1:** [5pts] (i) Let  $P = \langle p_0, \dots, p_{n-1} \rangle$  and  $Q = \langle q_0, \dots, q_{n-1} \rangle$  be two nonintersecting convex polygons in  $\mathbb{R}^2$ . Show that the common tangent, both inner and outer, can be computed in  $O(\log n)$  time. You can assume that the sequence of vertices of  $P$  (and  $Q$ ) is stored in an array.

**Problem 2:** [10pts] (i) Given a collection  $\mathcal{R}$  of “red” nonintersecting line segments and another collection  $\mathcal{B}$  of “blue” nonintersecting segments in  $\mathbb{R}^2$ , show that all red-blue intersections (intersections between a red segment and a blue segment) can be counted in time  $O(n \log^2 n)$ , where  $n = |\mathcal{R}| + |\mathcal{B}|$ . What is the space complexity of your algorithm? Improve the space complexity to  $O(n)$ .

**(Hint:** Use a segment tree on  $x$ -projections of segments and, at each node  $v$ , count intersections among the segments stored at  $v$ . You need to store some additional information at each node of the segment tree. Make sure that each intersection is counted exactly once.)

**Extra credit:** Improve the time complexity to  $O(n \log n)$ .

**Problem 3:** [10pts] Let  $S$  be a set of  $n$  points in the plane. A point  $p \in S$  is called *maximal* if there is no point  $q \neq p \in S$  such that  $x(p) \leq x(q)$  and  $y(p) \leq y(q)$ . Describe an  $O(n \log h)$  time algorithm to compute the maximal points of  $S$ , where  $h$  is the output size.

**Extra credit:** Describe an  $O(n \log h)$  time algorithm to compute the maxima of a set of  $n$  points in  $\mathbb{R}^3$ ;  $h$  is again the output size.

**Problem 4:** [10pts] Let  $S$  be a set of  $n$  pairwise disjoint segments in  $\mathbb{R}^2$ , and let  $p$  be a point not lying on any segment of  $S$ . Describe an  $O(n \log n)$  algorithm for computing all segments that are visible from  $p$  — a segment  $e$  is *visible* from  $p$  if there is a point  $q \in e$  such that the segment  $pq$  does not intersect the interior of any other segment. **(Hint:** Rotate a ray around the point  $p$ .)

**Problem 5:** [15pts] Let  $P$  be a set of  $n$  points in  $\mathbb{R}^2$ . We define a map  $N : \mathbb{S}^1 \rightarrow P$ , where  $N(u) = \arg \max_{p \in P} \langle p, u \rangle$ .  $N$  induces a subdivision  $P^*$  of  $\mathbb{S}^1$ . What are the vertices and edges of  $P^*$ , and how fast can  $P^*$  be computed? ( $\langle p, u \rangle$  is the inner product of the two vectors)

We call a pair of vertices  $p, q \in P$  *antipodal* if there are two parallel lines  $h_p, h_q$  passing through  $p$  and  $q$ , respectively, so that  $P$  lies between them. The *width* of  $P$  is the minimum width of a strip that contains  $P$ . Show that a strip that realizes the width of  $P$  contains an edge of  $CH(P)$  and an antipodal pair. Describe an  $O(n \log n)$  algorithm for computing the width of  $P$ .

Describe an  $O(n \log n)$  algorithm for computing the minimum-area rectangle (of arbitrary orientation) that contains  $P$ .