CPS 102: Discrete Mathematics	**Instructor:** Bruce Maggs
Assignment 2	**Due:** Friday September 28, 2007

1 Inductive Reasoning (30 points)

Write a computer program to draw a two-dimensional space filling curve. Turn in your code and a sample of your output.

2 Methods of Proof (5 points)

Prove that the square of an even number is an even number using

(a) a direct proof.

(b) an indirect proof.

(c) a proof by contradiction.

3 Methods of Proof (5 points)

Prove that if n is an integer and $3n + 2$ is even, then n is even using

(a) an indirect proof.

(b) a proof by contradiction.

4 Methods of Proof (5 points)

Use a proof by cases to show that $\lfloor n/2 \rfloor \lfloor n/2 \rfloor = \lfloor n^2/4 \rfloor$ for all integers n.

5 Methods of Proof (5 points)

Prove or disprove that given a positive integer n, there are n consecutive odd positive integers that are primes.

6 Mathematical Induction (5 points)

Find a formula for

$$\frac{1}{1.2} + \frac{1}{2.3} + \ldots + \frac{1}{n(n+1)}$$

by examining the values of this expression for small values of n. Use mathematical induction to prove your result.

7 Mathematical Induction (5 points)

Prove that $1.1! + 2.2! + \ldots + n.n! = (n+1)! - 1$ whenever n is a positive integer.

8 Mathematical Induction (5 points)

Use mathematical induction to prove that

$$1.2.3 + 2.3.4 + \ldots + n(n+1)(n+2) = n(n+1)(n+2)(n+3)/4$$

9 Mathematical Induction (5 points)

Prove that

$$1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

whenever n is a positive integer greater than 1.

10 Mathematical Induction (10 points)

Use mathematical induction to prove that a set with n elements has $n(n-1)(n-2)/6$ subsets containing exactly three elements whenever n is an integer greater than or equal to 3.

11 Mathematical Induction (10 points)

Prove that if $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets such that $A_k \subseteq B_k$ for $k = 1, 2, \ldots, n$, then

(a) $\bigcup_{k=1}^{n} A_k \subseteq \bigcup_{k=1}^{n} B_k$

(b) $\bigcap_{k=1}^{n} A_k \subseteq \bigcap_{k=1}^{n} B_k$

12 Mathematical Induction (5 points)

Use the formula for the sum of the terms of a geometric progression to evaluate the following sums.

(a) $4 + 4.3 + 4.3^2 + \ldots + 4.3^8$

(b) $3 + 3.2^2 + 3.2^4 + \ldots + 3.2^{10}$

(c) $1 - 2 + 2^2 - 2^3 + \ldots + (-1)^n 2^n$

13 Mathematical Induction (10 points)

Find the flaw with the following “proof” that $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.
BASIC STEP: $a^0 = 1$ is true by the definition of a^0.
INDUCTIVE STEP: Assume that $a^k = 1$ for all nonnegative integers k with $k \leq n$. Then note that

$$a^{n+1} = \frac{a^n.a^n}{a^{n-1}} = \frac{1.1}{1} = 1$$

14 Mathematical Induction (10 points)

Show that the following form of mathematical induction is a valid method to prove that $P(n)$ is true for all positive integers n.
BASIC STEP: $P(1)$ and $P(2)$ are true.
INDUCTIVE STEP: For each positive integer n, if $P(n)$ and $P(n+1)$ are both true, then $P(n+2)$ is true.

15 Recursive Definitions (5 points)

Find $f(1)$, $f(2)$, $f(3)$, $f(4)$ and $f(5)$ if $f(n)$ is defined recursively by $f(0) = 3$ and for $n = 0, 1, 2, \ldots$

(a) $f(n+1) = -2f(n)$

(b) $f(n+1) = 3f(n) + 7$

(c) $f(n+1) = f(n)^2 - 2f(n) - 2$

(d) $f(n+1) = 3^{f(n)/3}$

16 Recursive Definitions (5 points)

Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3 \ldots$ if

(a) $a_n = 4n - 2$

(b) $a_n = 1 + (-1)^n$

(c) $a_n = n(n + 1)$

(d) $a_n = n^2$

17 Recursive Definitions (5 points)

Show that each of the following proposed recursive definitions of a function on the set of positive integers does not produce a well-defined function

(a) $F(n) = 1 + F(\lfloor n/2 \rfloor)$ for $n \geq 1$ and $F(1) = 1$

(b) $F(n) = 1 + F(n - 3)$ for $n \geq 2$, $F(1) = 2$ and $F(2) = 3$

(c) $F(n) = 1 + F(n/2)$ for $n \geq 2$, $F(1) = 1$ and $F(2) = 2$

(d) $F(n) = 1 + F(n/2)$ if n is even and $n \geq 2$, $F(n) = 1 - F(n - 1)$ if n is odd, and $F(1) = 1$

(e) $F(n) = 1 + F(n/2)$ if n is even and $n \geq 2$, $F(n) = F(3n - 1)$ if n is odd and $n \geq 3$, $F(1) = 1$