

Relational Database Design Theory

CPS 116
Introduction to Database Systems

Announcements (Tue. Sep. 8) ²

- ❖ Homework #1 due in one week
 - Help session this Friday or next Monday?
 - Friday (Sep. 11): 4-5pm?
 - Next Monday (Sep. 14): 4-5pm?
 - Will email the announcement
- ❖ Course project description available on Thursday

Motivation ³

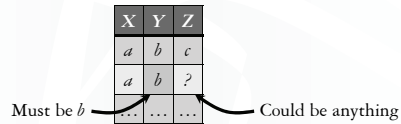
<i>SID</i>	<i>name</i>	<i>CID</i>
142	Bart	CPS116
142	Bart	CPS114
857	Lisa	CPS116
857	Lisa	CPS130
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- ❖ How do we tell if a design is bad, e.g., *StudentEnroll* (*SID*, *name*, *CID*)?
- ❖ How about a systematic approach to detecting and removing redundancy in designs?
 - Dependencies, decompositions, and normal forms

Functional dependencies

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- ❖ A functional dependency (FD) has the form $X \rightarrow Y$, where X and Y are sets of attributes in a relation R
- ❖ $X \rightarrow Y$ means that whenever two tuples in R agree on all the attributes in X , they must also agree on all attributes in Y



FD examples

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Address (*street_address*, *city*, *state*, *zip*)

- ❖ $street_address, city, state \rightarrow zip$

- Trivial FD: $LHS \supseteq RHS$
- Completely non-trivial FD: $LHS \cap RHS = \emptyset$

Keys redefined using FD's

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A set of attributes K is a key for a relation R if

- ❖ $K \rightarrow$ all (other) attributes of R
 - That is, K is a "super key"
- ❖ No proper subset of K satisfies the above condition
 - That is, K is minimal

Reasoning with FD's

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Given a relation R and a set of FD's \mathcal{F}

- ❖ Does another FD follow from \mathcal{F} ?
 - Are some of the FD's in \mathcal{F} redundant (i.e., they follow from the others)?
- ❖ Is K a key of R ?
 - What are all the keys of R ?

Attribute closure

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❖ Given R , a set of FD's \mathcal{F} that hold in R , and a set of attributes Z in R :

The closure of Z (denoted Z^+) with respect to \mathcal{F} is the set of all attributes $\{A_1, A_2, \dots\}$ functionally determined by Z (that is, $Z \rightarrow A_1 A_2 \dots$)

- ❖ Algorithm for computing the closure
 - Start with closure = Z
 - If $X \rightarrow Y$ is in \mathcal{F} and X is already in the closure, then also add Y to the closure
 - Repeat until no more attributes can be added

A more complex example

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StudentGrade (*SID*, *name*, *email*, *CID*, *grade*)

(Not a good design, and we will see why later)

Example of computing closure

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- ❖ \mathcal{F} includes:
 - $SID \rightarrow name, email$
 - $email \rightarrow SID$
 - $SID, CID \rightarrow grade$
- ❖ $\{ CID, email \}^+ = ?$
- ❖ $email \rightarrow SID$
 - Add SID ; closure is now $\{ CID, email, SID \}$
- ❖ $SID \rightarrow name, email$
 - Add $name, email$; closure is now $\{ CID, email, SID, name \}$
- ❖ $SID, CID \rightarrow grade$
 - Add $grade$; closure is now all the attributes in *StudentGrade*

Using attribute closure

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Given a relation R and set of FD's \mathcal{F}

- ❖ Does another FD $X \rightarrow Y$ follow from \mathcal{F} ?
 - Compute X^+ with respect to \mathcal{F}
 - If $Y \subseteq X^+$, then $X \rightarrow Y$ follow from \mathcal{F}
- ❖ Is K a key of R ?
 - Compute K^+ with respect to \mathcal{F}
 - If K^+ contains all the attributes of R , K is a super key
 - Still need to verify that K is *minimal* (how?)

Rules of FD's

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- ❖ Armstrong's axioms
 - Reflexivity: If $Y \subseteq X$, then $X \rightarrow Y$
 - Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
 - Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- ❖ Rules derived from axioms
 - Splitting: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
 - Combining: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
- ☞ Using these rules, you can prove or disprove an FD given a set of FDs

Non-key FD's

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- ❖ Consider a non-trivial FD $X \rightarrow Y$ where X is not a super key
 - Since X is not a super key, there are some attributes (say Z) that are not functionally determined by X

X	Y	Z
a	b	c_1
a	b	c_2
...

That b is always associated with a is recorded by multiple rows:
 redundancy, update anomaly, deletion anomaly

Example of redundancy

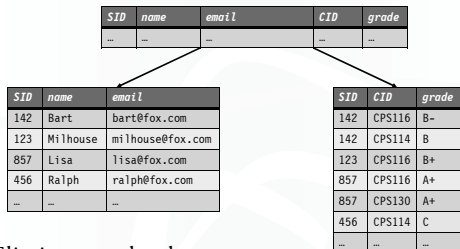
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- ❖ *StudentGrade* (SID , $name$, $email$, CID , $grade$)
- ❖ $SID \rightarrow name, email$

SID	$name$	$email$	CID	$grade$
142	Bart	bart@fox.com	CPS116	B-
142	Bart	bart@fox.com	CPS114	B
123	Milhouse	milhouse@fox.com	CPS116	B+
857	Lisa	lisa@fox.com	CPS116	A+
857	Lisa	lisa@fox.com	CPS130	A+
456	Ralph	ralph@fox.com	CPS114	C
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Decomposition

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- ❖ Eliminates redundancy
- ❖ To get back to the original relation:

Unnecessary decomposition

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<i>SID</i>	<i>name</i>	<i>email</i>
142	Bart	bart@fox.com
123	Milhouse	milhouse@fox.com
857	Lisa	lisa@fox.com
456	Ralph	ralph@fox.com
...

<i>SID</i>	<i>name</i>
142	Bart
123	Milhouse
857	Lisa
456	Ralph
...	...

<i>SID</i>	<i>email</i>
142	bart@fox.com
123	milhouse@fox.com
857	lisa@fox.com
456	ralph@fox.com
...	...

- ❖ Fine: join returns the original relation
- ❖ Unnecessary: no redundancy is removed, and now *SID* is stored twice!

Bad decomposition

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<i>SID</i>	<i>CID</i>	<i>grade</i>
142	CPS116	B-
142	CPS114	B
123	CPS116	B+
857	CPS116	A+
857	CPS130	A+
456	CPS114	C
...

<i>SID</i>	<i>CID</i>
142	CPS116
142	CPS114
123	CPS116
857	CPS116
857	CPS130
456	CPS114
...	...

<i>SID</i>	<i>grade</i>
142	B-
142	B
123	B+
857	A+
456	C
...	...

- ❖ Association between *CID* and *grade* is lost
- ❖ Join returns more rows than the original relation

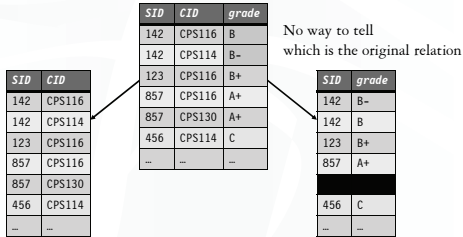
Lossless join decomposition

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- ❖ Decompose relation *R* into relations *S* and *T*
 - $attrs(R) = attrs(S) \cup attrs(T)$
 - $S = \pi_{attrs(S)}(R)$
 - $T = \pi_{attrs(T)}(R)$
- ❖ The decomposition is a lossless join decomposition if, given known constraints such as FD's, we can guarantee that $R = S \bowtie T$
- ❖ Any decomposition gives $R \subseteq S \bowtie T$ (why?)
 - A lossy decomposition is one with $R \subset S \bowtie T$

Loss? But I got more rows!

- ❖ “Loss” refers not to the loss of tuples, but to the loss of information
 - Or, the ability to distinguish different original relations



Questions about decomposition

- ❖ When to decompose
- ❖ How to come up with a correct decomposition (i.e., lossless join decomposition)

An answer: BCNF

- ❖ A relation R is in Boyce-Codd Normal Form if
 - For every non-trivial FD $X \rightarrow Y$ in R , X is a super key
 - That is, all FDs follow from “key \rightarrow other attributes”
- ❖ When to decompose
 - As long as some relation is not in BCNF
- ❖ How to come up with a correct decomposition
 - Always decompose on a BCNF violation (details next)
 - ☞ Then it is guaranteed to be a lossless join decomposition!

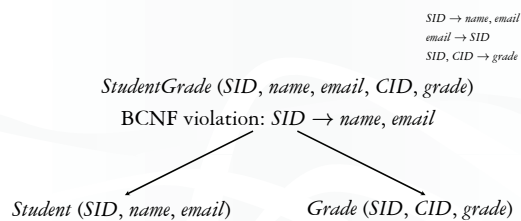
BCNF decomposition algorithm

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- ❖ Find a BCNF violation
 - That is, a non-trivial FD $X \rightarrow Y$ in R where X is not a super key of R
- ❖ Decompose R into R_1 and R_2 , where
 - R_1 has attributes $X \cup Y$
 - R_2 has attributes $X \cup Z$, where Z contains all attributes of R that are in neither X nor Y
- ❖ Repeat until all relations are in BCNF

BCNF decomposition example

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Another example

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SID → *name, email*
email → *SID*
SID, CID → *grade*

StudentGrade (*SID, name, email, CID, grade*)

BCNF violation:

Why is BCNF decomposition lossless ²⁵

Given non-trivial $X \rightarrow Y$ in R where X is not a super key of R , need to prove:

- ❖ Anything we project always comes back in the join:
 - $R \subseteq \pi_{XY}(R) \bowtie \pi_{XZ}(R)$
 - Sure; and it doesn't depend on the FD
- ❖ Anything that comes back in the join must be in the original relation:
 - $R \supseteq \pi_{XY}(R) \bowtie \pi_{XZ}(R)$
 - Proof makes use of the fact that $X \rightarrow Y$

Recap ²⁶

- ❖ Functional dependencies: a generalization of the key concept
- ❖ Non-key functional dependencies: a source of redundancy
- ❖ BCNF decomposition: a method for removing redundancies
 - BNCf decomposition is a lossless join decomposition
- ❖ BCNF: schema in this normal form has no redundancy due to FD's

BCNF = no redundancy? ²⁷

❖ *Student (SID, CID, club)*

- Suppose your classes have nothing to do with the clubs you join
- FD's?
- BCNF?
- Redundancies?

SID	CID	club
142	CPS116	ballet
142	CPS116	sumo
142	CPS114	ballet
142	CPS114	sumo
123	CPS116	chess
123	CPS116	golf
...

Multivalued dependencies

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- ❖ A multivalued dependency (MVD) has the form $X \twoheadrightarrow Y$, where X and Y are sets of attributes in a relation R
- ❖ $X \twoheadrightarrow Y$ means that whenever two rows in R agree on all the attributes of X , then we can swap their Y components and get two new rows that are also in R

X	Y	Z
a	b1	c1
a	b2	c2
a	b1	c2
a	b2	c1
...

} Must be in R too

MVD examples

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Student (SID, CID, club)

- ❖ $SID \twoheadrightarrow CID$
- ❖ $SID, CID \twoheadrightarrow club$
- ❖ $SID, CID \twoheadrightarrow SID$

Complete MVD + FD rules

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- ❖ FD reflexivity, augmentation, and transitivity
- ❖ MVD complementation:
If $X \twoheadrightarrow Y$, then $X \twoheadrightarrow attr_3(R) - X - Y$
- ❖ MVD augmentation:
If $X \twoheadrightarrow Y$ and $V \subseteq W$, then $XW \twoheadrightarrow YV$
- ❖ MVD transitivity:
If $X \twoheadrightarrow Y$ and $Y \twoheadrightarrow Z$, then $X \twoheadrightarrow Z - Y$
- ❖ Replication (FD is MVD):
If $X \rightarrow Y$, then $X \twoheadrightarrow Y$ Try proving things using these!
- ❖ Coalescence:
If $X \twoheadrightarrow Y$ and $Z \subseteq Y$ and there is some W disjoint from Y such that $W \rightarrow Z$, then $X \rightarrow Z$

An elegant solution: chase

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❖ Given a set of FD's and MVD's \mathcal{D} , does another dependency d (FD or MVD) follow from \mathcal{D} ?

❖ Procedure

- Start with the hypothesis of d , and treat them as "seed" tuples in a relation
- Apply the given dependencies in \mathcal{D} repeatedly
 - If we apply an FD, we infer equality of two symbols
 - If we apply an MVD, we infer more tuples
- If we infer the conclusion of d , we have a proof
- Otherwise, if nothing more can be inferred, we have a counterexample

Proof by chase

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❖ In $R(A, B, C, D)$, does $A \twoheadrightarrow B$ and $B \twoheadrightarrow C$ imply that $A \twoheadrightarrow C$?

	Have	Need																								
	<table border="1" style="display: inline-table;"> <thead> <tr><th>A</th><th>B</th><th>C</th><th>D</th></tr> </thead> <tbody> <tr><td>a</td><td>b1</td><td>c1</td><td>d1</td></tr> <tr><td>a</td><td>b2</td><td>c2</td><td>d2</td></tr> </tbody> </table>	A	B	C	D	a	b1	c1	d1	a	b2	c2	d2	<table border="1" style="display: inline-table;"> <thead> <tr><th>A</th><th>B</th><th>C</th><th>D</th></tr> </thead> <tbody> <tr><td>a</td><td>b1</td><td>c2</td><td>d1</td></tr> <tr><td>a</td><td>b2</td><td>c1</td><td>d2</td></tr> </tbody> </table>	A	B	C	D	a	b1	c2	d1	a	b2	c1	d2
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$B \twoheadrightarrow C$	<table border="1" style="display: inline-table;"> <tbody> <tr><td>a</td><td>b2</td><td>c1</td><td>d2</td></tr> <tr><td>a</td><td>b2</td><td>c2</td><td>d1</td></tr> </tbody> </table>	a	b2	c1	d2	a	b2	c2	d1																	
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Another proof by chase

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❖ In $R(A, B, C, D)$, does $A \rightarrow B$ and $B \rightarrow C$ imply that $A \rightarrow C$?

	Have	Need												
	<table border="1" style="display: inline-table;"> <thead> <tr><th>A</th><th>B</th><th>C</th><th>D</th></tr> </thead> <tbody> <tr><td>a</td><td>b1</td><td>c1</td><td>d1</td></tr> <tr><td>a</td><td>b2</td><td>c2</td><td>d2</td></tr> </tbody> </table>	A	B	C	D	a	b1	c1	d1	a	b2	c2	d2	$c1 = c2$ \neq
A	B	C	D											
a	b1	c1	d1											
a	b2	c2	d2											
$A \rightarrow B$	$b1 = b2$													
$B \rightarrow C$	$c1 = c2$													

In general, both new tuples and new equalities may be generated

Counterexample by chase

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❖ In $R(A, B, C, D)$, does $A \twoheadrightarrow BC$ and $CD \rightarrow B$ imply that $A \rightarrow B$?

	Have				Need
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	$b_1 = b_2$ ❖
$A \twoheadrightarrow BC$	<i>a</i>	<i>b</i> ₁	<i>c</i> ₁	<i>d</i> ₁	
	<i>a</i>	<i>b</i> ₂	<i>c</i> ₂	<i>d</i> ₂	
	<i>a</i>	<i>b</i> ₂	<i>c</i> ₂	<i>d</i> ₁	
	<i>a</i>	<i>b</i> ₁	<i>c</i> ₁	<i>d</i> ₂	

Counterexample!

4NF

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- ❖ A relation R is in Fourth Normal Form (4NF) if
 - For every non-trivial MVD $X \twoheadrightarrow Y$ in R , X is a superkey
 - That is, all FD's and MVD's follow from "key \rightarrow other attributes" (i.e., no MVD's and no FD's besides key functional dependencies)
- ❖ 4NF is stronger than BCNF
 - Because every FD is also a MVD

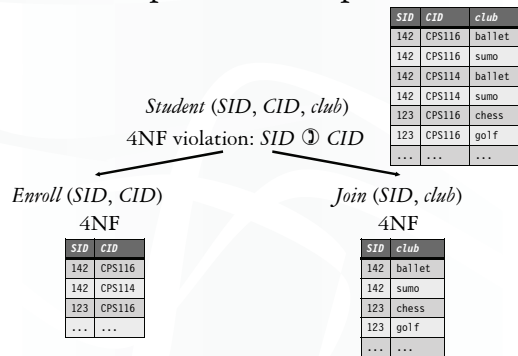
4NF decomposition algorithm

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- ❖ Find a 4NF violation
 - A non-trivial MVD $X \twoheadrightarrow Y$ in R where X is not a superkey
- ❖ Decompose R into R_1 and R_2 , where
 - R_1 has attributes $X \cup Y$
 - R_2 has attributes $X \cup Z$ (Z contains attributes not in X or Y)
- ❖ Repeat until all relations are in 4NF
- ❖ Almost identical to BCNF decomposition algorithm
- ❖ Any decomposition on a 4NF violation is lossless

4NF decomposition example

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Summary

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- ❖ Philosophy behind BCNF, 4NF:
Data should depend on the key, the whole key, and nothing but the key!
- ❖ Other normal forms
 - 3NF: More relaxed than BCNF; will not remove redundancy if doing so makes FDs harder to enforce
 - 2NF: Slightly more relaxed than 3NF
 - 1NF: All column values must be atomic
