The Clarke (aka. VCG) mechanism [Clarke 71]

- The Clarke mechanism chooses some outcome $o$ that maximizes $\Sigma_i v_i(\theta_i', o)$
  - $\theta_i' = \text{the type that } i \text{ reports}$
- To determine the payment that agent $j$ must make:
  - Pretend $j$ does not exist, and choose $o_{-j}$ that maximizes $\Sigma_{i \neq j} v_i(\theta_i', o_{-j})$
  - $j$ pays $\Sigma_{i \neq j} v_i(\theta_i', o_{-j}) - \Sigma_{i \neq j} v_i(\theta_i', o)$
The Clarke mechanism is strategy-proof

- Total utility for agent $j$ is
  \[ v_j(\theta_j, o) + \sum_{i \neq j} v_i(\theta_i', o) - \sum_{i \neq j} v_i(\theta_i', o_j) \]
- But agent $j$ cannot affect the choice of $o_j$
- Hence, $j$ can focus on maximizing $v_j(\theta_j, o) + \sum_{i \neq j} v_i(\theta_i', o)$
  - The other agents’ types $\theta_i'$ are out of $j$’s control
  - Agent $j$’s true type $\theta_j$ is also fixed
  - Agent $j$ will try to report a type $\theta_j'$ so that $o$ maximizes $v_j(\theta_j, o) + \sum_{i \neq j} v_i(\theta_i', o)$
- But mechanism chooses $o$ to maximize $\sum_i v_i(\theta_i', o)$
- Hence, if $\theta_j' = \theta_j$, $o$ maximizes $\sum_i v_i(\theta_i', o) = v_j(\theta_j', o) + \sum_{i \neq j} v_i(\theta_i', o)$. Therefore, if $\theta_j' = \theta_j$, $o$ maximizes $v_j(\theta_j, o) + \sum_{i \neq j} v_i(\theta_i', o)$. 