CPS 102: Discrete Mathematics	**Instructor:** Bruce Maggs
Assignment 2	**Due:** Monday, October 3th, 2011

1 (15 points)

Prove that 1/3 is a recurring decimal number (in other words, it does not have a finite decimal representation). *Hint:* Use induction on the number of digits in any representation.

2 (4 points)

Prove that if n is an integer and $3n + 2$ is even, then n is even using

- an indirect proof (an indirect proof proves an argument of the form, if p is *true* then q is *true*, by proving the *contrapositive* - if q is *false*, then p is *false*)
- a proof by contradiction

3 (6 points)

Prove that except for 3, 5, 7, no three consecutive odd positive integers are prime.

4 (15 points)

Prove that $\sqrt{2}$ is an irrational number. *Hint*: Any rational number can be represented using integers p and q, in the form $\frac{p}{q}$ such that $q \neq 0$ and with $GCD(p, q) = 1$.

5 (5 points)

Prove that there are infinitely many primes.

6 (5 points)

Prove or disprove: If a, b, and m are positive integers, then $(a \bmod m) + (b \bmod m) = (a + b) \bmod m$

7 (3 points)

Using mathematical induction prove that $1 \cdot 1! + 2 \cdot 2! + ... + n \cdot n! = (n+1)! - 1$ whenever n is a positive integer.

8 (3 points)

Use mathematical induction to prove that $1{\cdot}2{\cdot}3{+}2{\cdot}3{\cdot}4{+}...{+}n{\cdot}(n{+}1){\cdot}(n{+}2) = n(n{+}1)(n{+}2)(n{+}3)/4$

9 (4 points)

Use mathematical induction to prove that a set with n elements has $n(n-1)(n-2)/6$ subsets containing exactly three elements whenever n is an integer greater than or equal to 3

10 (5 points)

Find the flaw with the following "proof" that $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.
Basic Step: $a^0 = 1$ is true by the definition of a^0.
Inductive Step: Assume that $a^k = 1$ for all nonnegative integers k with $k \leq n$. Then note that –
$a^{n+1} = \frac{a^n \cdot a^n}{a^{n-1}} = \frac{1 \cdot 1}{1} = 1$

11 (15 points)

Prove that for any integer n a $2^n \times 2^n$ grid with *one corner square removed* can be covered with L shaped tiles made up of 3 squares.